

Zero-Group-Velocity Lamb modes in Anisotropic Plates

Daniel A. Kiefer, Sylvain Mezil, François Bruno, Claire Prada

Institut Langevin, ESPCI Paris, Université PSL, CNRS, Paris, France



Institut **Langevin**
ONDES ET IMAGES





Laser impact on a plate induces narrow local resonances

Duralumin plate

Thickness $d = 1.5$ mm

**Superimposed
source
and probe**

**Nd:YAG
Pulsed Laser**

**Heterodyne
Interferometer**

1064 nm

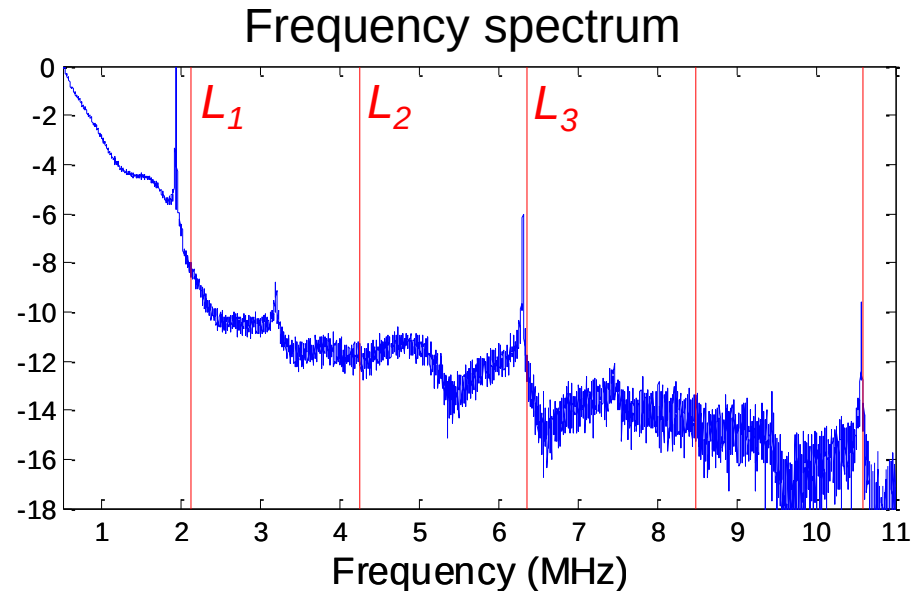
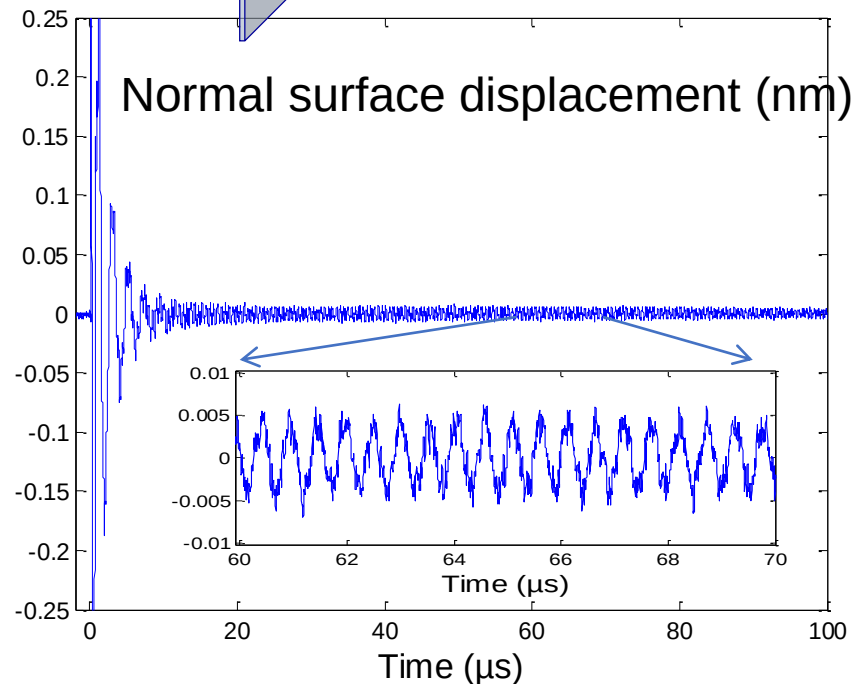
Pulse duration: 8 ns

Energy: 0.1-40 mJ

Thermoelastic regime

532 nm

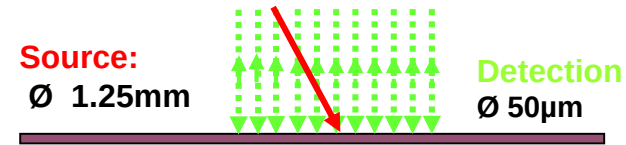
BW: 10 kHz - 45 MHz – 120 mV/nm



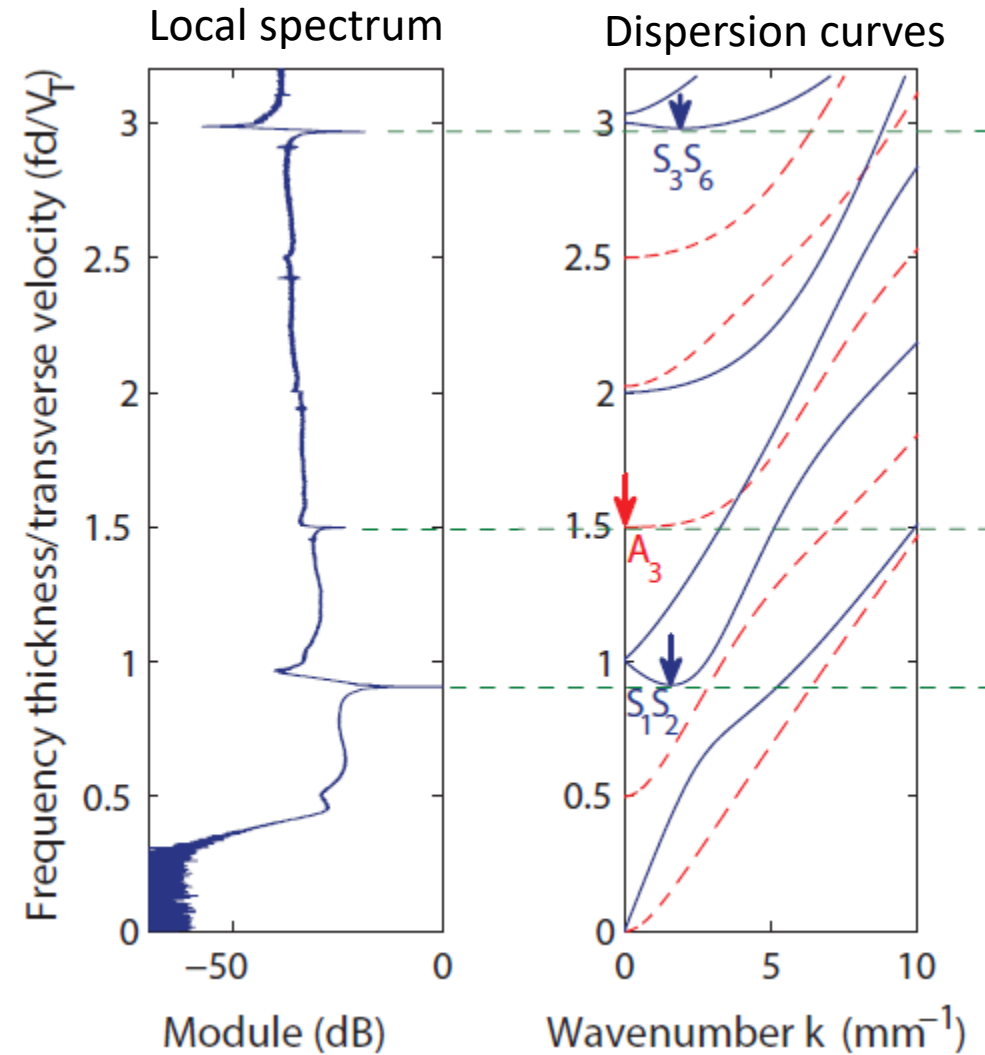
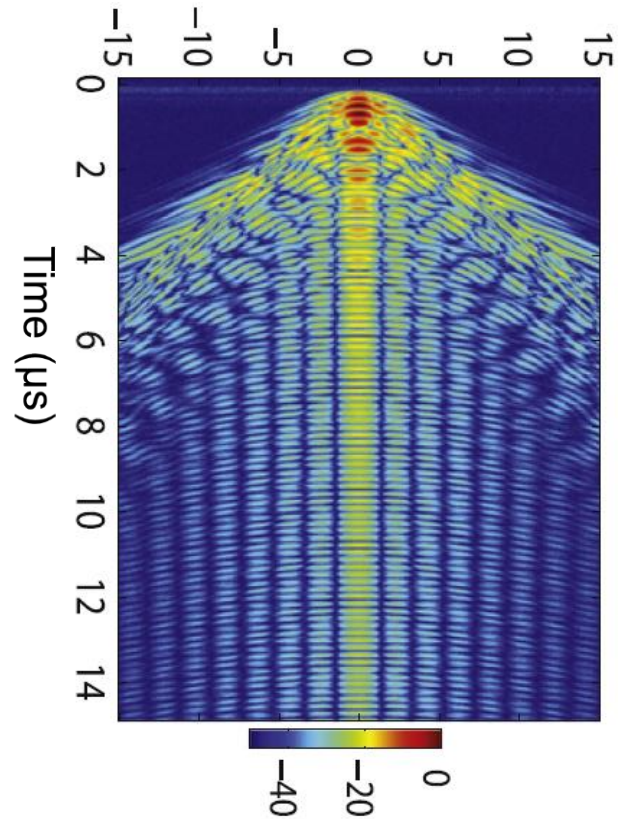
Longitudinal thickness
resonance frequencies
 $L_n = nV_L/2d$ ($V_L = 6.14$ mm/ μ s)



Resonances and dispersion curves in a duraluminum plate



Normal surface displacement



$\frac{d\omega}{dk} = 0$
Zero Group Velocity
↓
Energy is trapped
under the source



Laser impact induced resonances : spatial distribution

Normal surface displacement $u(r,t)$



Temporal Fourier Transform $u(r,f)$

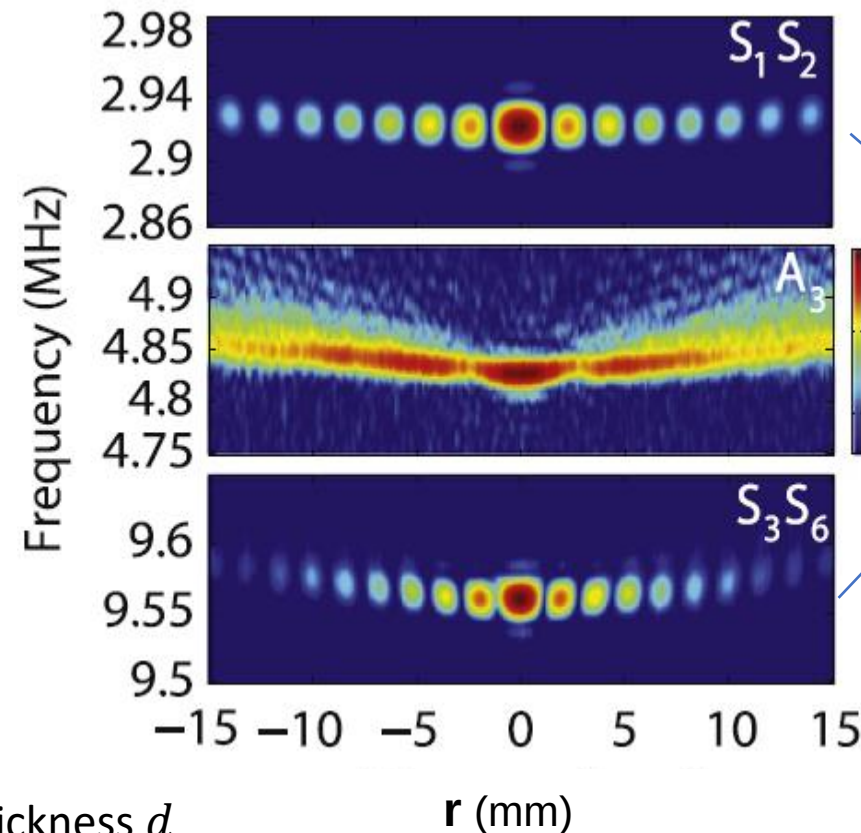
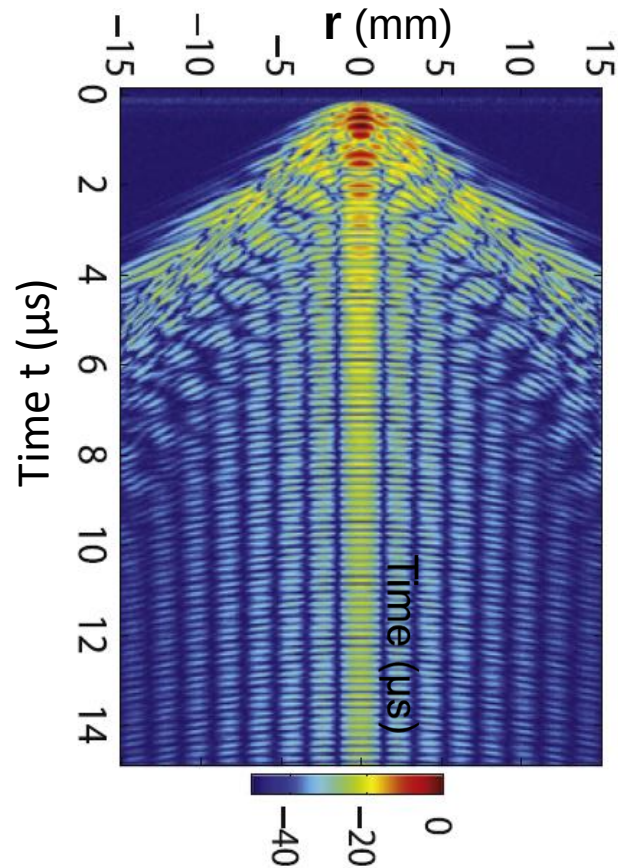


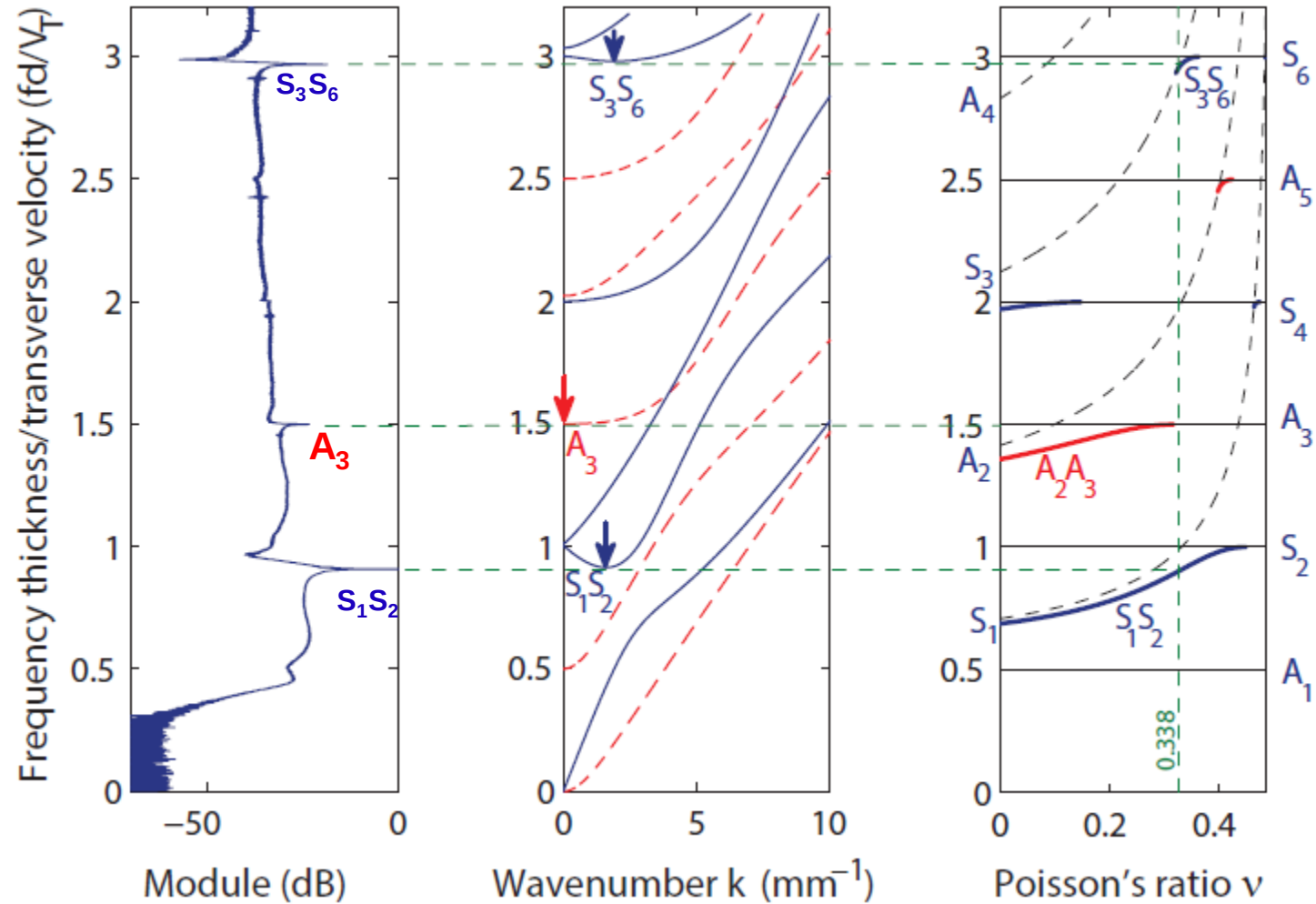
Plate thickness d

Wavelength $\lambda \sim 3d$

Optimal Gaussian source diameter to excite a ZGV mode $D = \frac{\lambda}{\pi}$

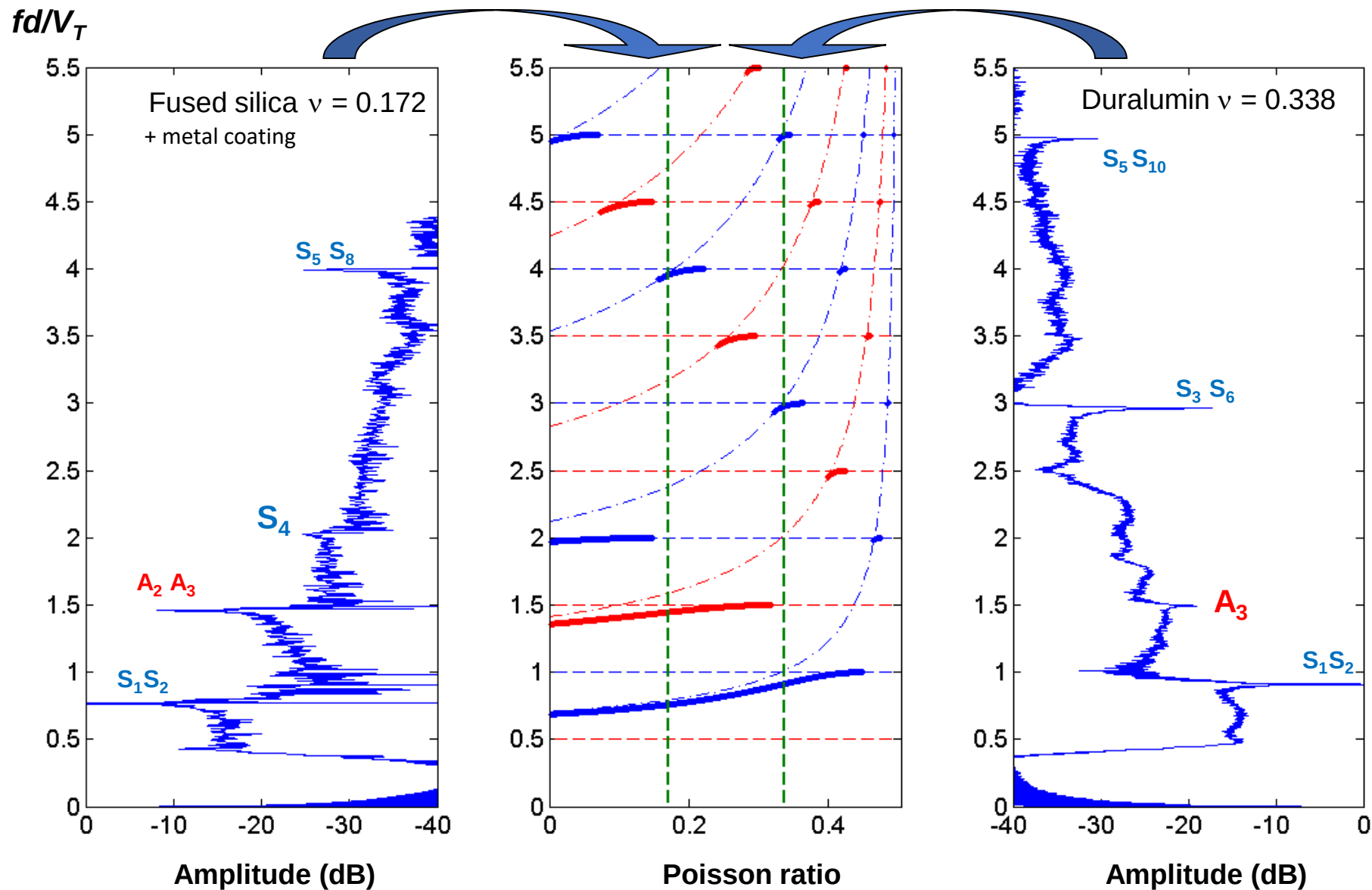


Laser impact generated resonances and Poisson's ratio





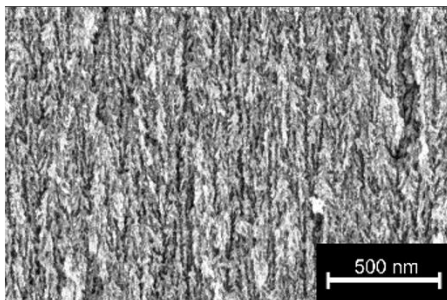
Application : Non contact Poisson's ratio measurement



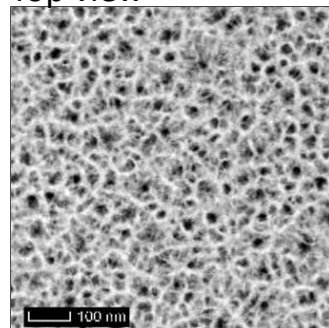
Application : effective Poisson's ratio of nanoporous silicon

Scanning electron microscopy

Side view



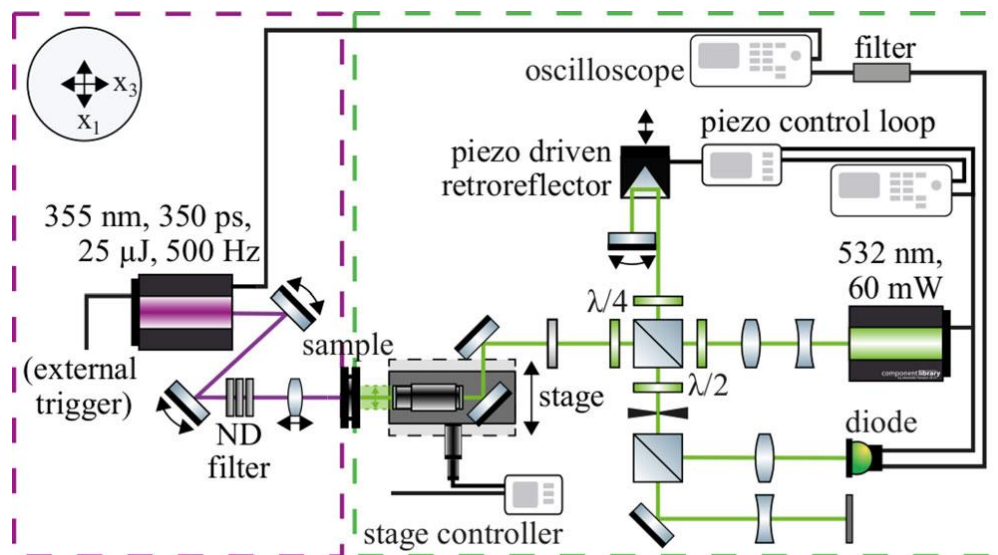
Top view



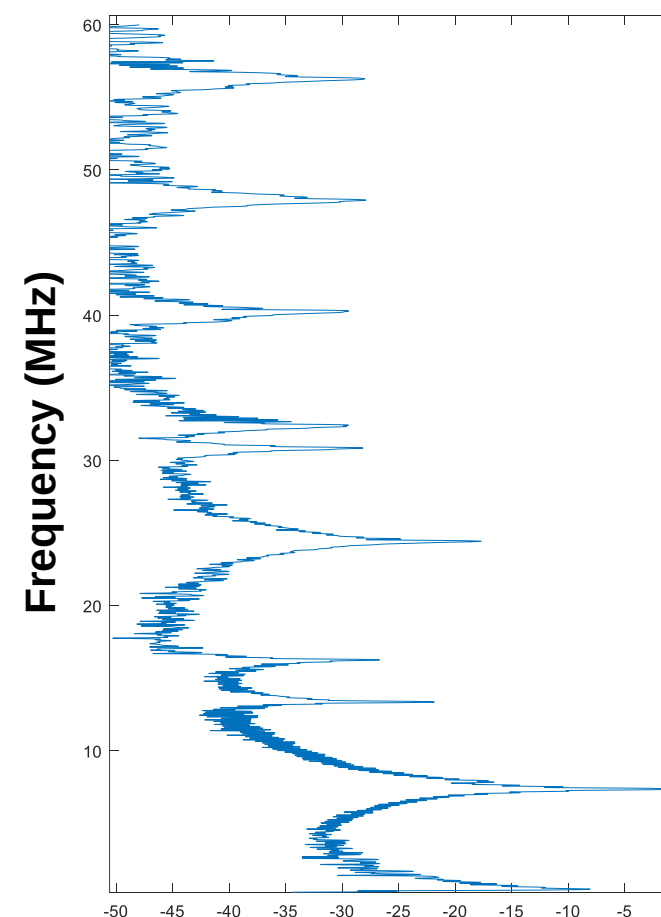
Porosity 55%

Mean pore radius 3.7 nm

Membrane thickness 250 μm



ZGV and thickness resonances



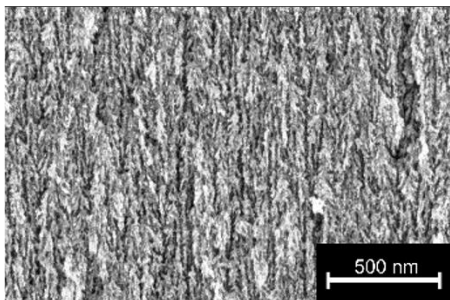
300 μm thick sample

20 nm silver coating for the detection

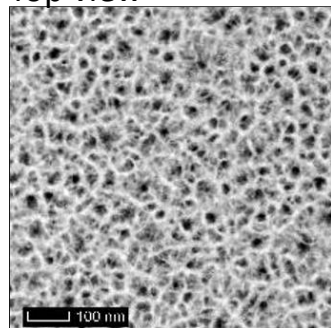
Application : effective Poisson's ratio of nanoporous silicon

Scanning electron microscopy

Side view



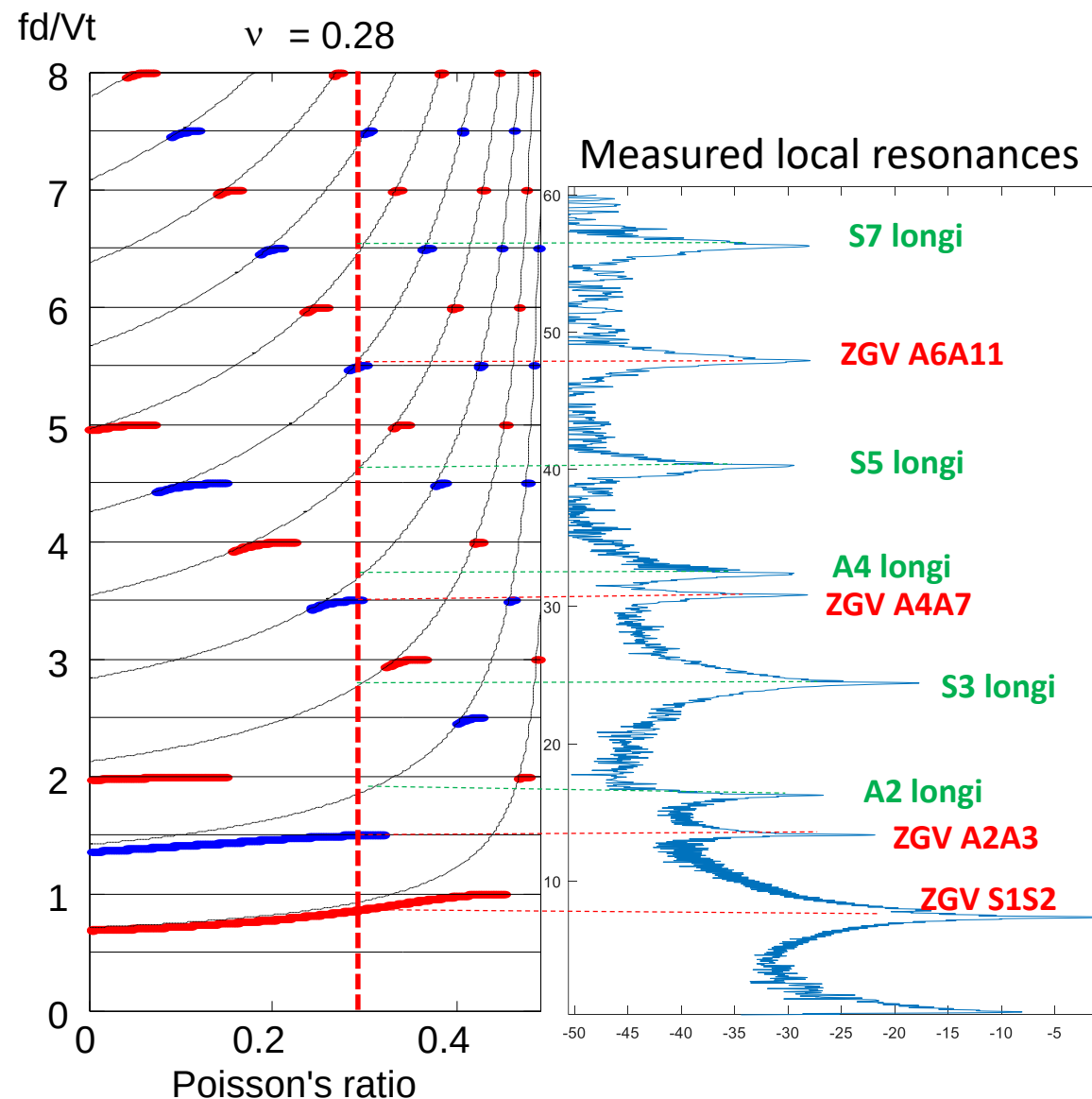
Top view



Porosity 55%
Mean pore radius 3.7 nm
Membrane thickness 250 μm

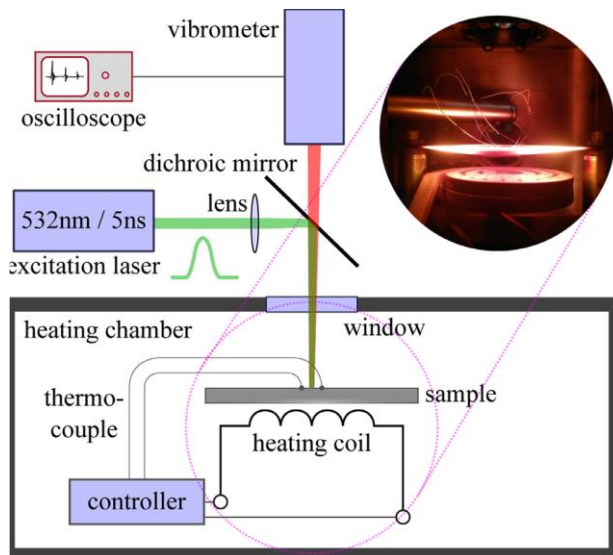
Provide an estimation of some elastic constants

$V_L = 5.54 \text{ mm}/\mu\text{s}$ and $V_T = 3.26 \text{ mm}/\mu\text{s}$ through thickness





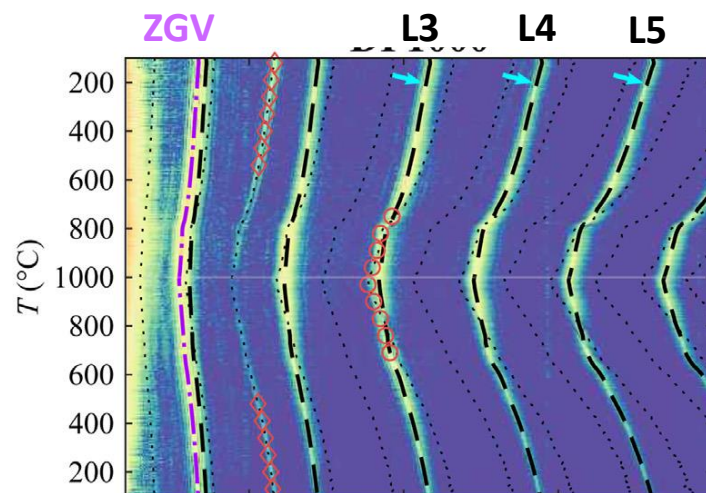
Application : Poisson's ratio measured along thermal cycles



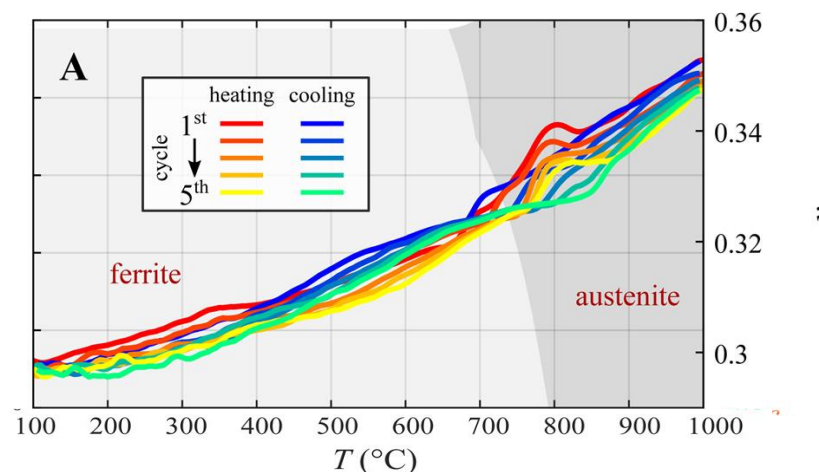
Local measurement in reflection
on 0.8 x 50 x 50 mm steel plates

Excitation slightly above ablation threshold
in order to excite both ZGV and longitudinal resonances

Local resonance spectrum along thermal cycles



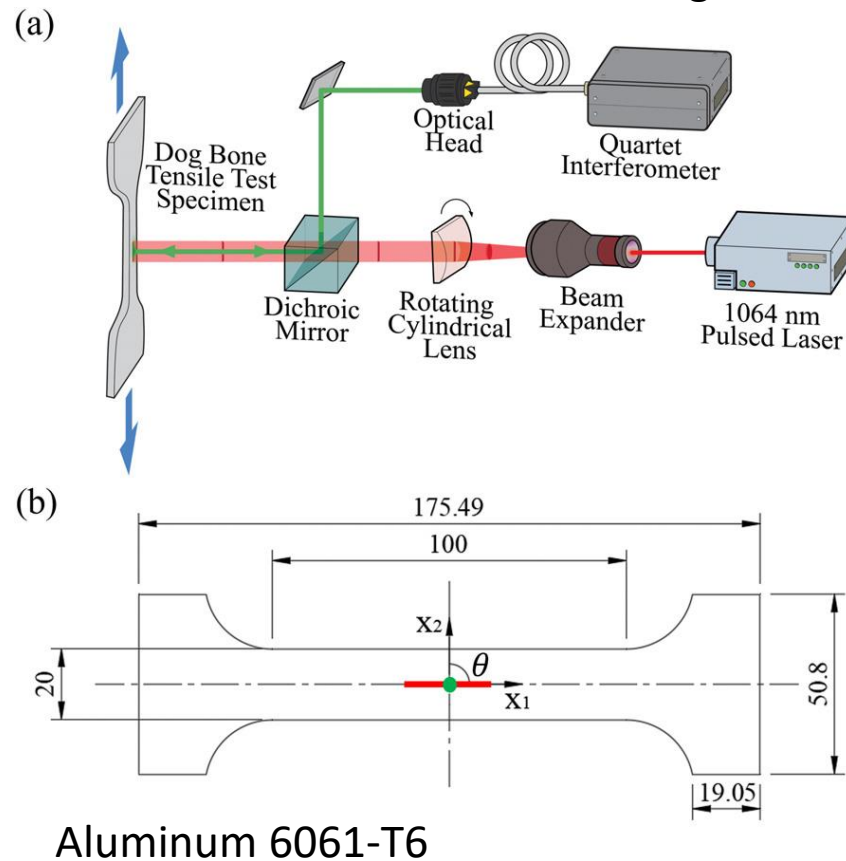
Poisson's ratio deduced from f_{L3}/f_{ZGV}



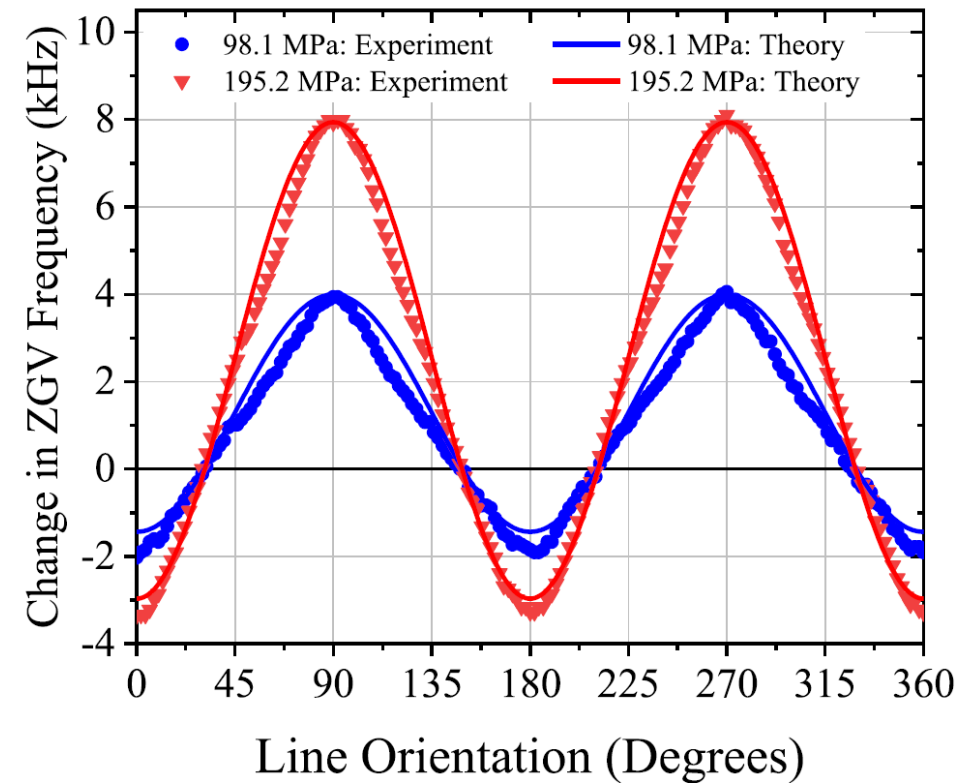
In situ laser-ultrasonic monitoring of Poisson's ratio and bulk sound velocities of steel plates during thermal processes,
Watzl,..., Grunsteidl, Acta Materialia 235 (2022)



ZGV modes are measured using a line source



Strain induced anisotropy





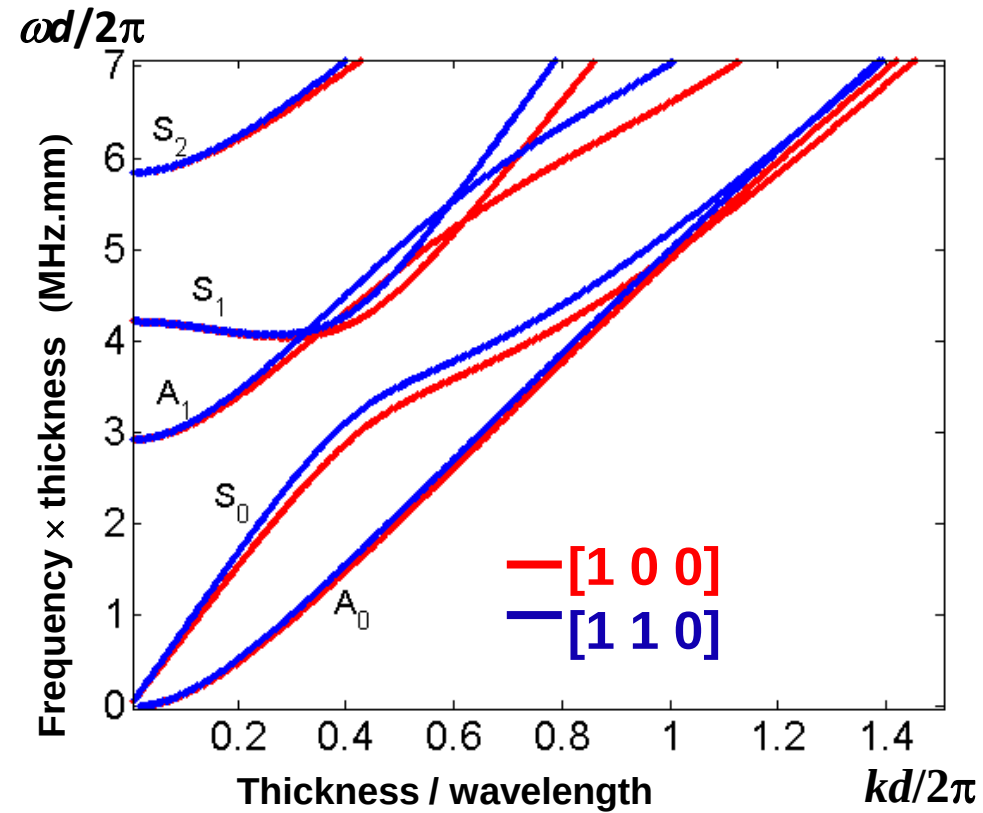
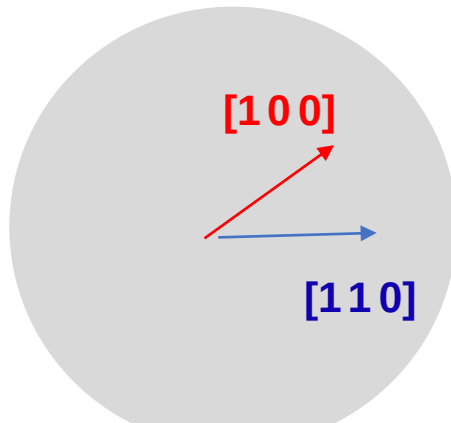
ZGV modes in anisotropic plates : silicon plate

Silicon wafer

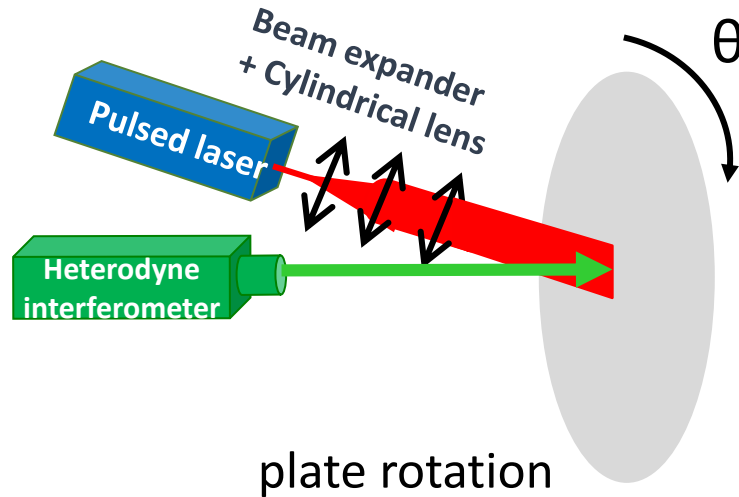
cut: [0 0 1]

thickness: 525 μ m

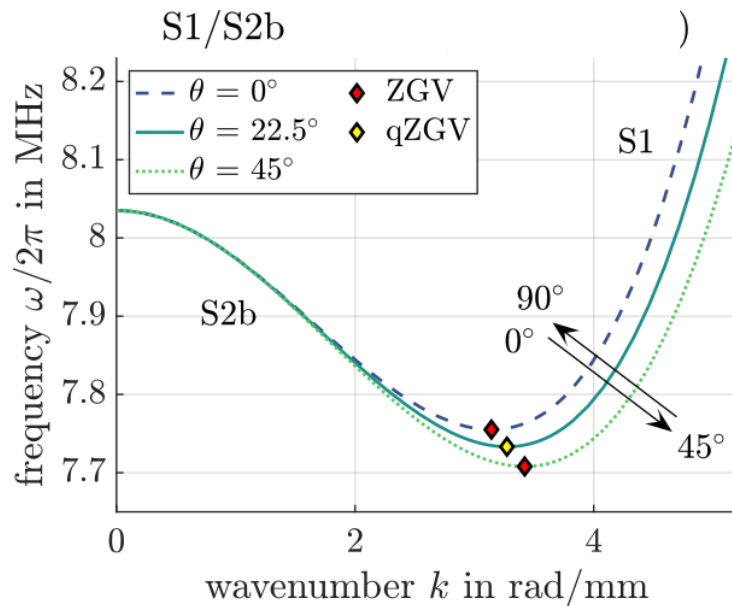
diameter: 5 ''



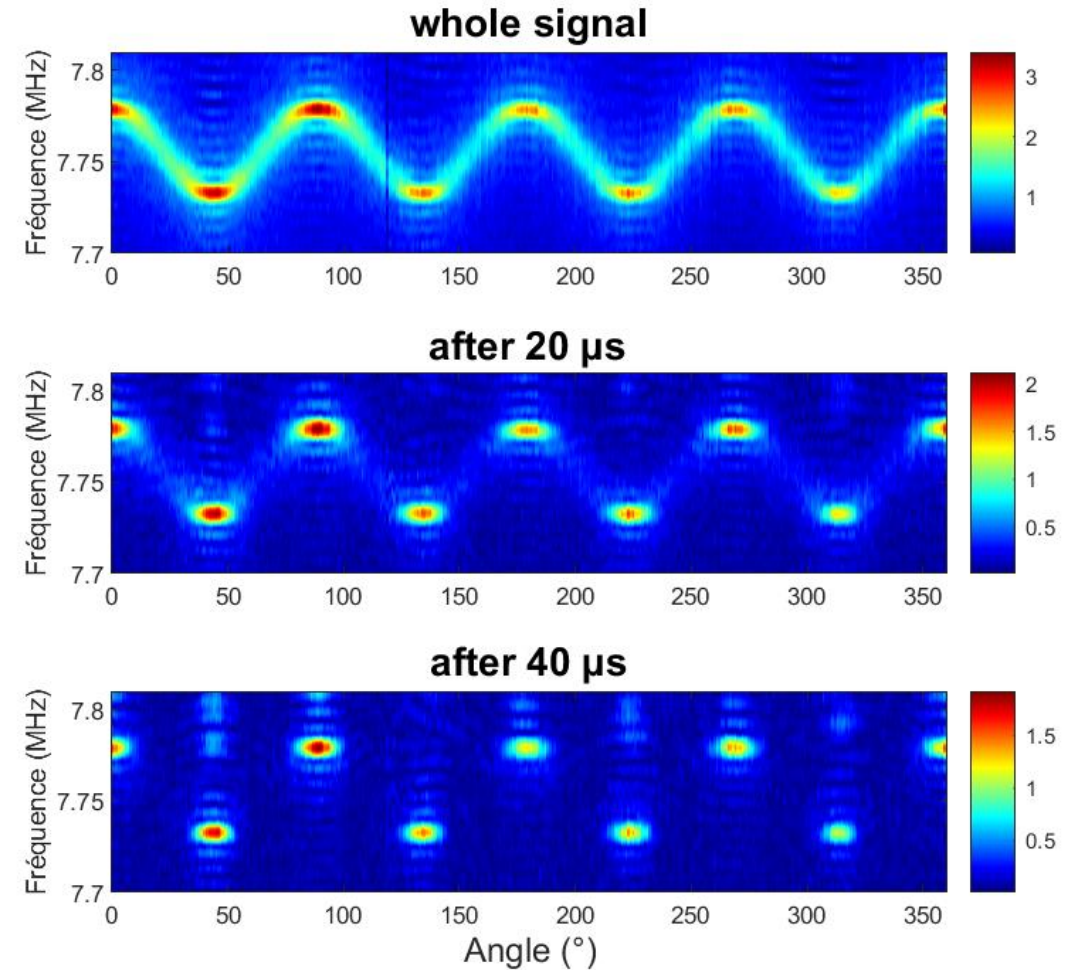
Line source excitation of ZGV mode in a silicon wafer



0° 110
45° 100



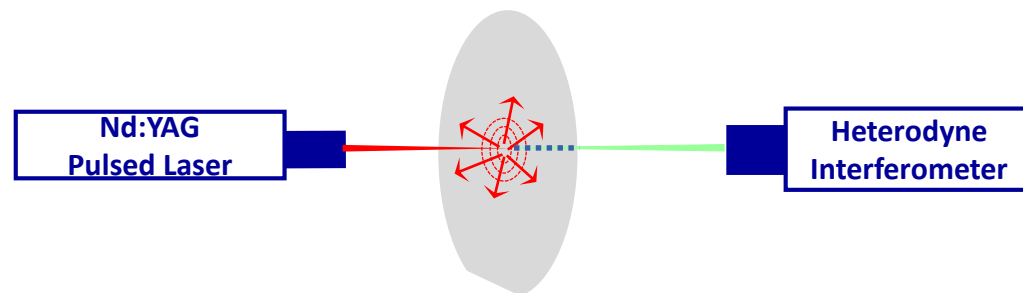
Angle dependence of $S_1 S_2$ ZGV mode



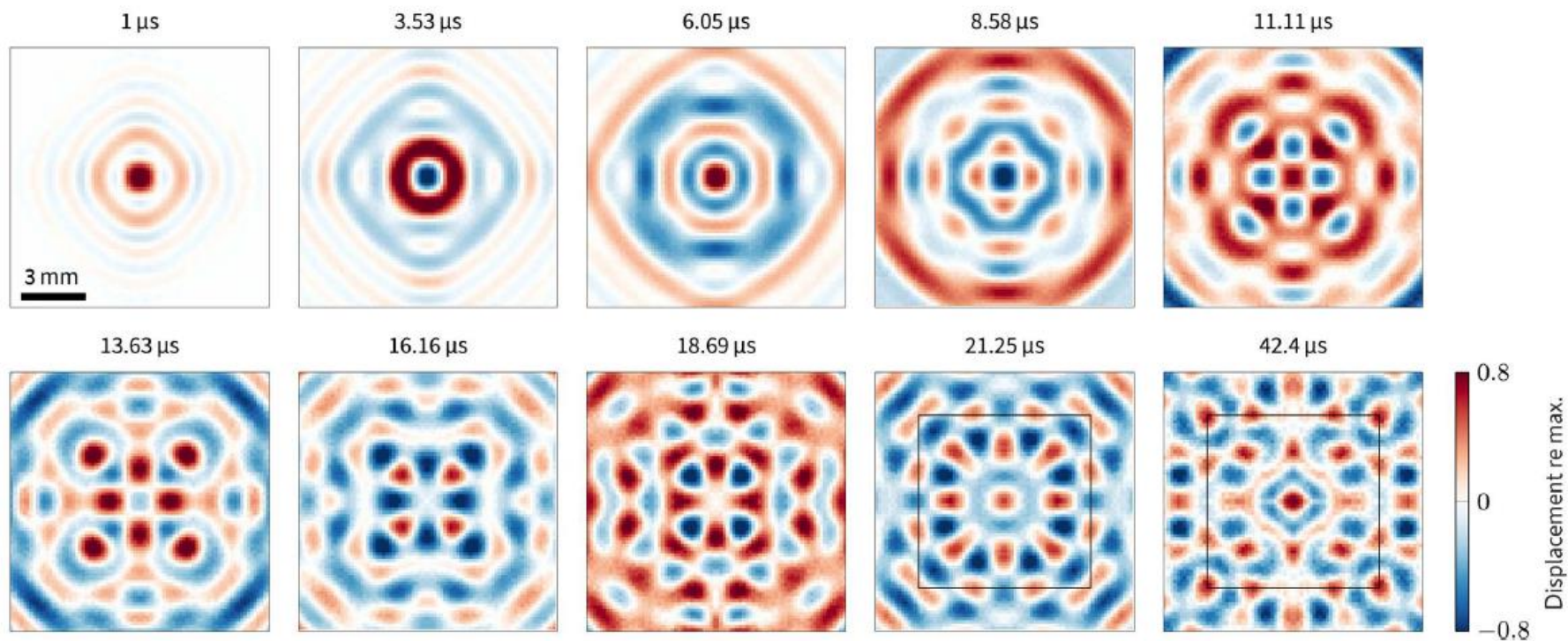


Point source excitation in a silicon plate

2D scan
of the source

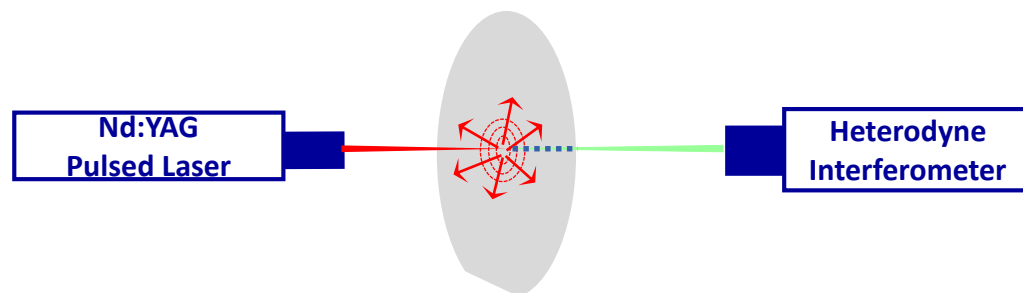


Normal surface displacement

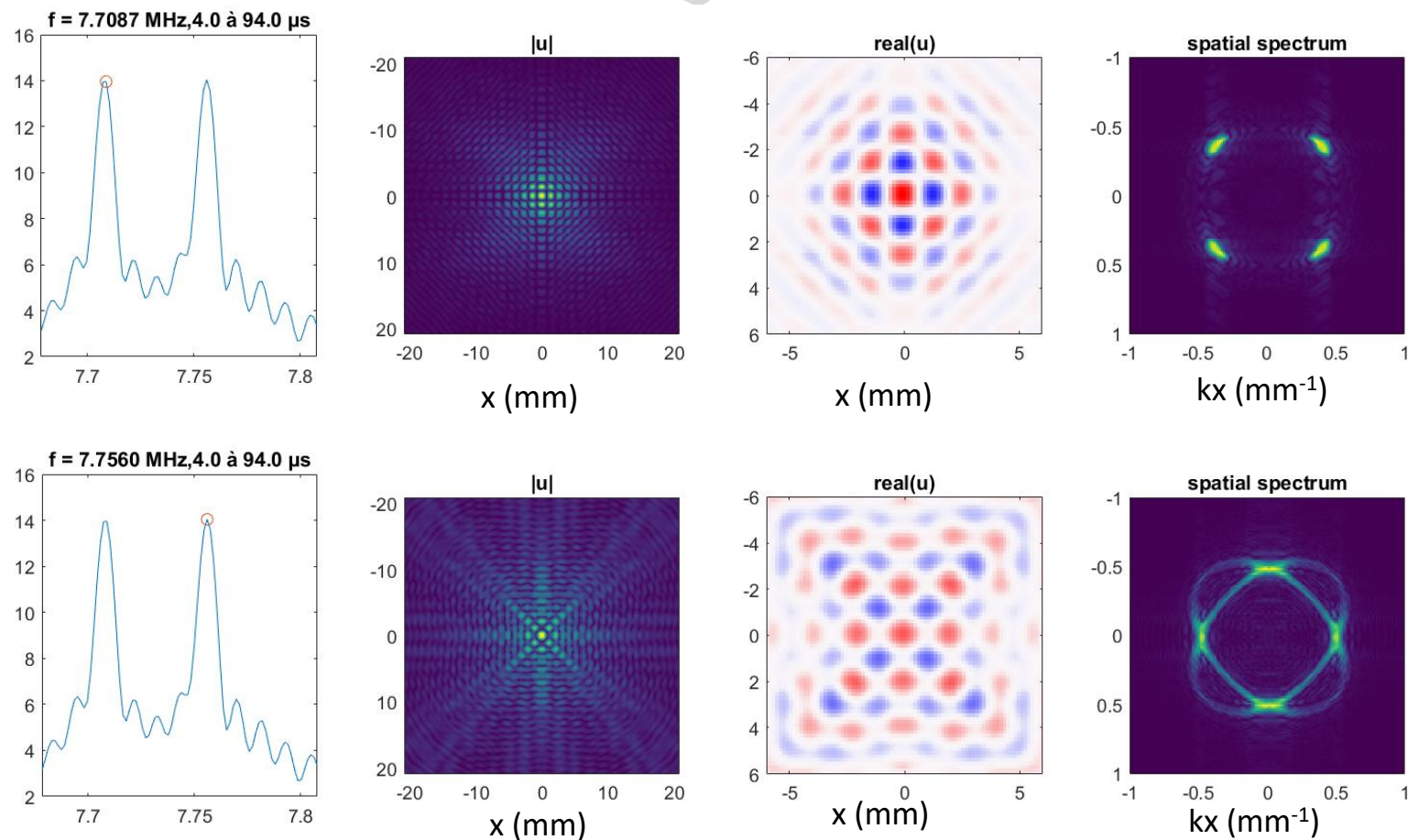




ZGV modes excited with a point source in a silicon plate

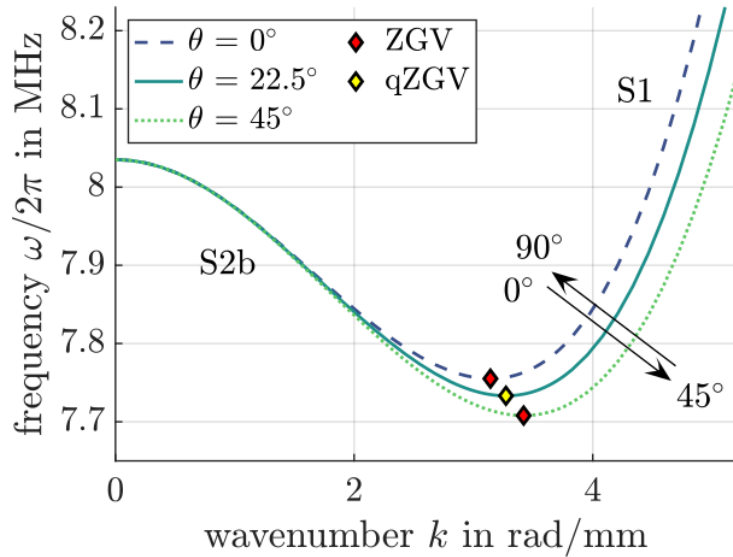


Local spectrum

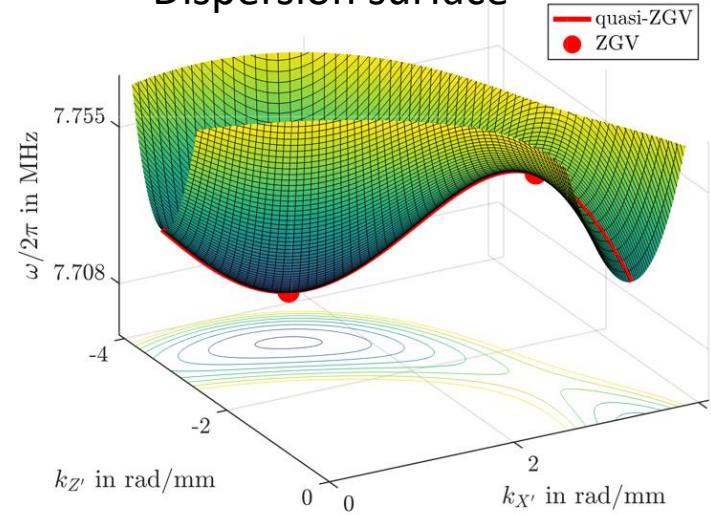


Dispersion surface and ZGV modes in a cubic silicon plate

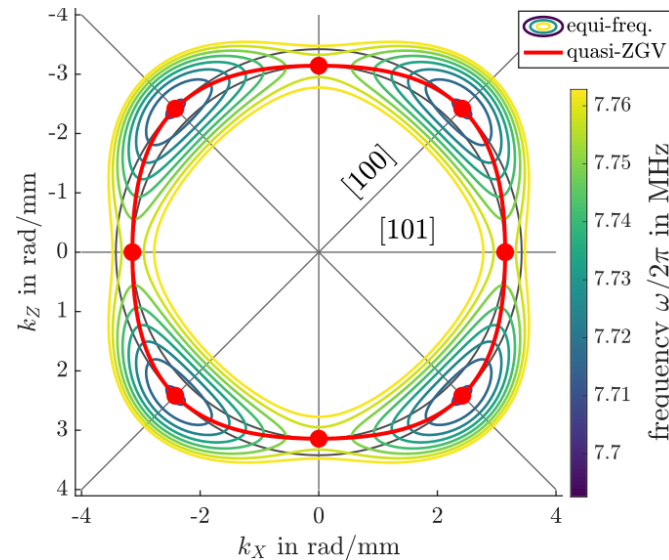
Dispersion curves



Dispersion surface



Isofrequency contours



The group velocity

$$V_g(\omega, \theta) = \frac{\partial \omega}{\partial k} e_r + \frac{1}{k} \frac{\partial \omega}{\partial \theta} e_\theta$$

$V_g(\omega, \theta) = 0$ only on principal axes

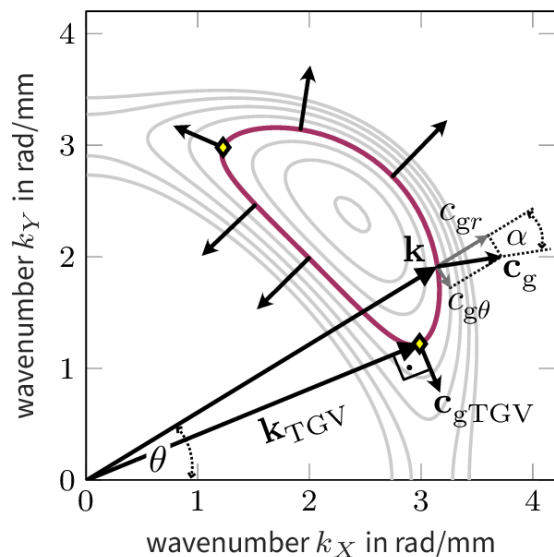
- 4 minima at $45^\circ + n 90^\circ$ ($n=0-3$)
- 4 saddle points at $n 90^\circ$
- **8 ZGV points**

Calculated with GEW dispersion script (2022)

https://github.com/dakiefer/GEW_dispersion_script



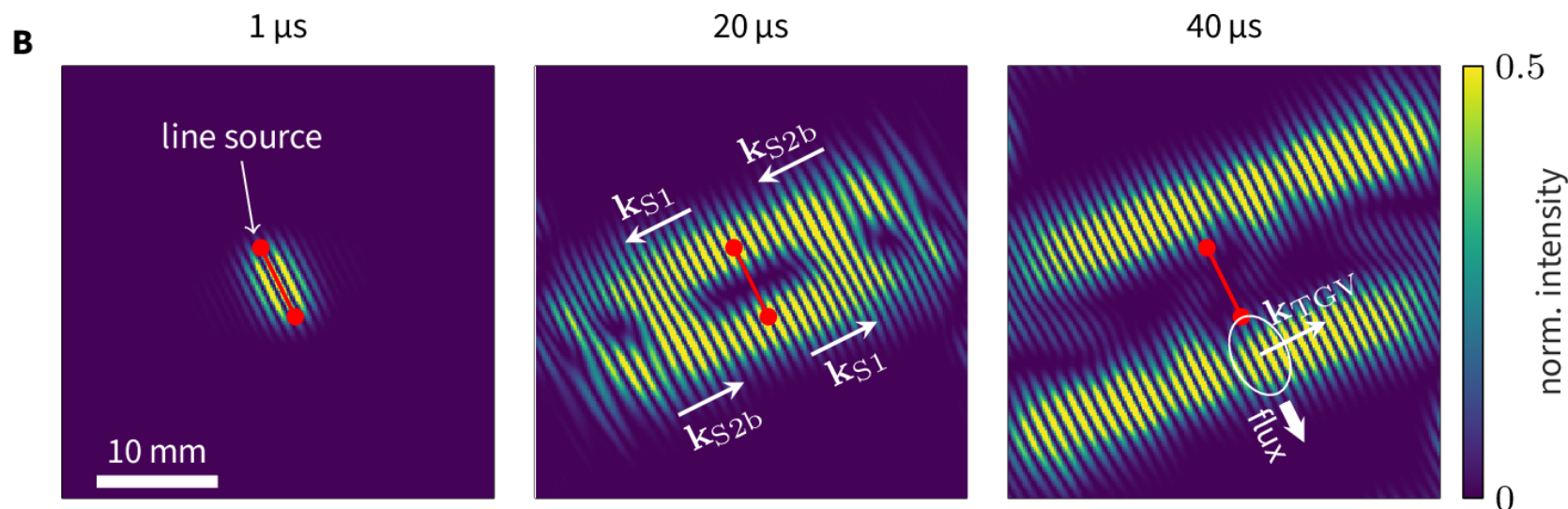
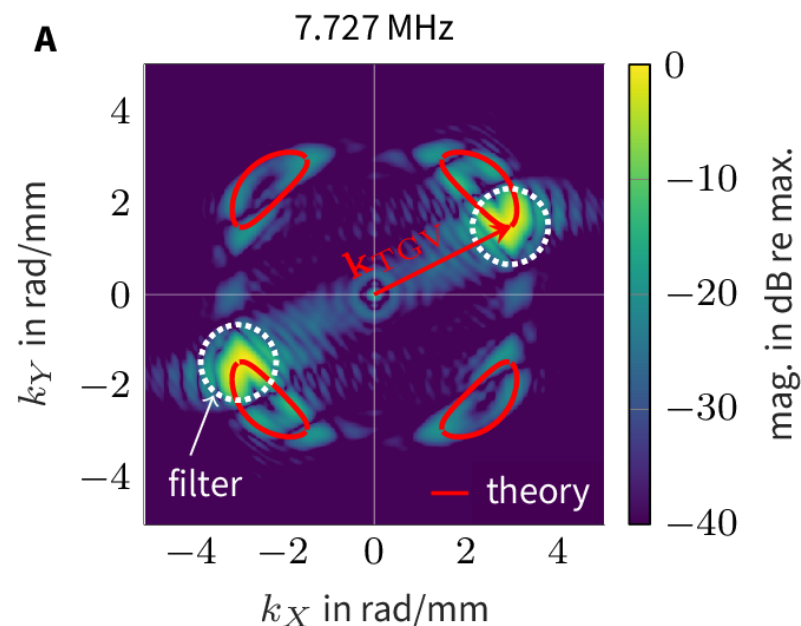
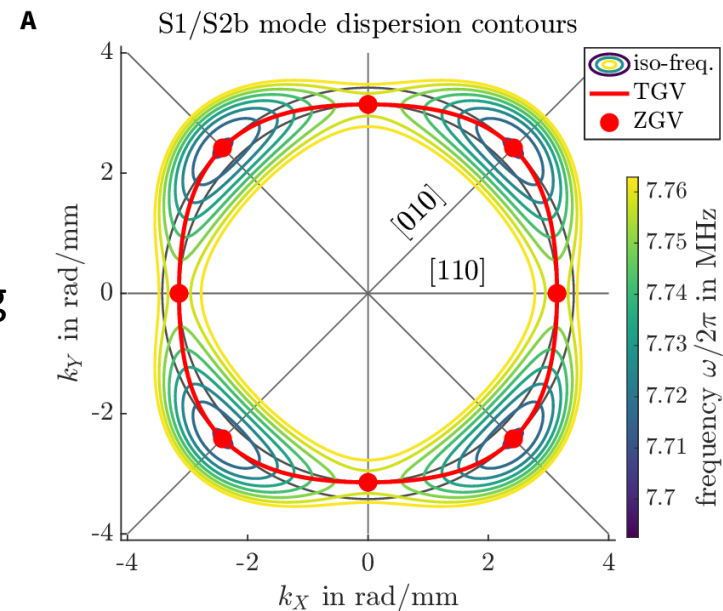
ZGV and Transverse Group Velocity (TGV) modes



The group velocity vector $\mathbf{C}_g$ is orthogonal to the isofrequency contours

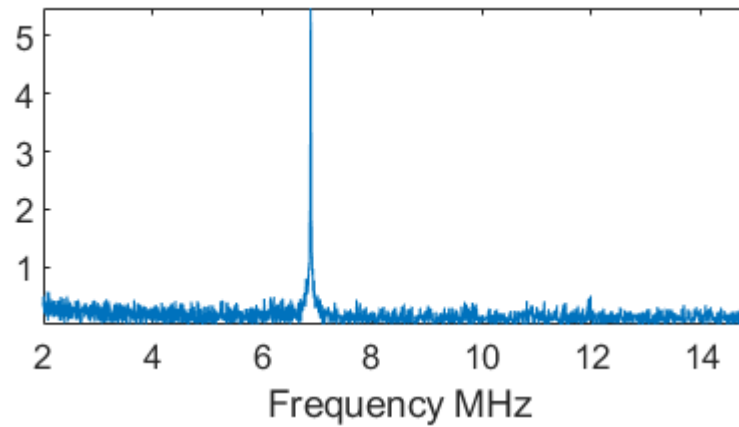
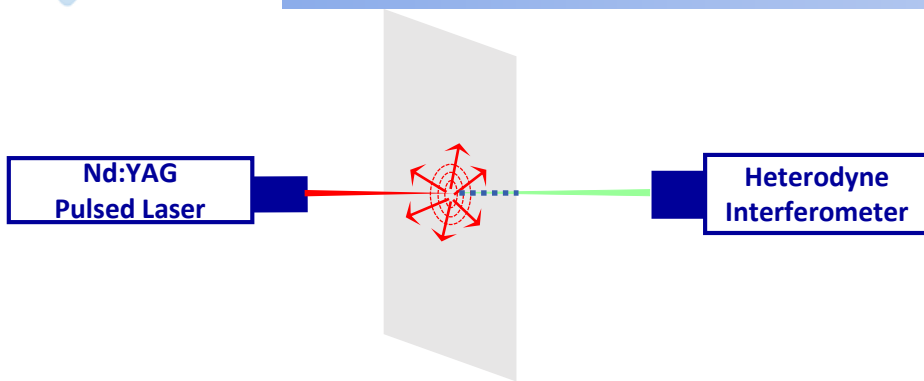
Squewing angle α between wavevector $\mathbf{k}$ and $\mathbf{C}_g$

$\alpha = 90^\circ >$ Transverse Group Velocity mode





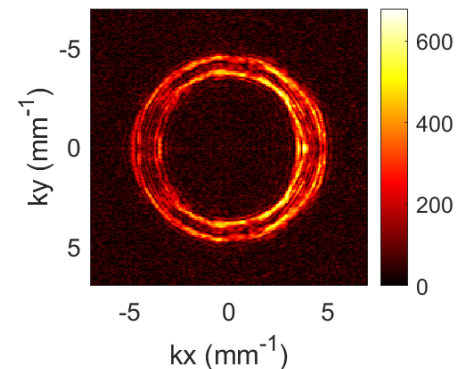
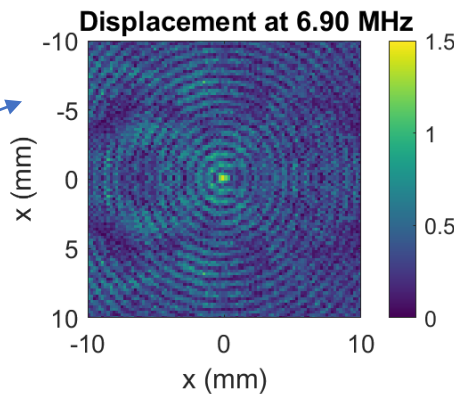
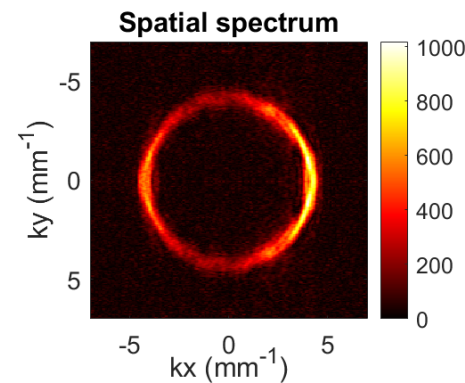
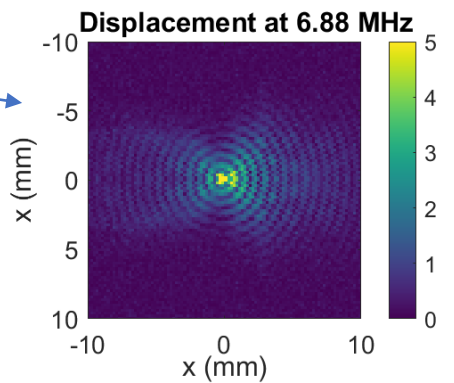
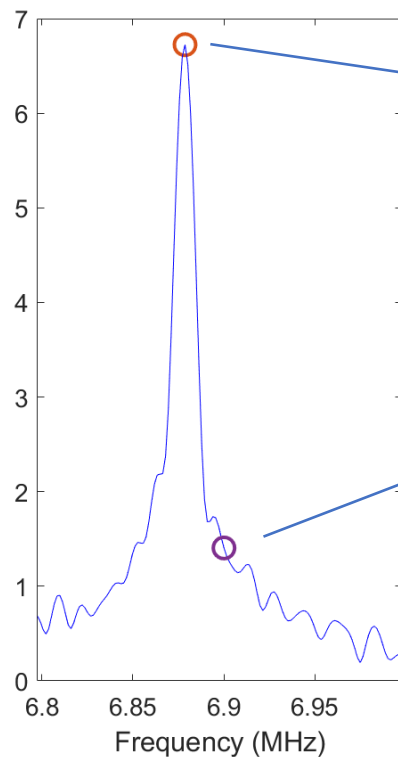
ZGV resonance measured in a cold rolled carbon steel plates



Thickness-gauge-strip
50x300mm ; 400 μ m
Carbonsteel 1.1274
Cold rolled, hardened

Chemische Analyse:
C = 0,95 – 1,05 %
Si = 0,10 – 0,30 %
Mn = 0,30 – 0,50 %
P = max. 0,02 %
S = max. 0,03 %

hasberg
SCHNEIDER GMBH



Single resonance
Isotropy

ZGV measured in a cold rolled stainless steel plates

Thickness-gauge-strip
100x500mm ; 500 μm
Steel 1.4310 (301)
Cold rolled, hardened

Chemische Analyse:

C = max. 0,15 %

Si = max. 2,00 %

Mn = max. 2,00 %

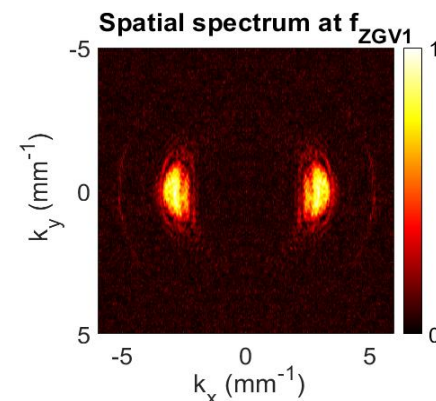
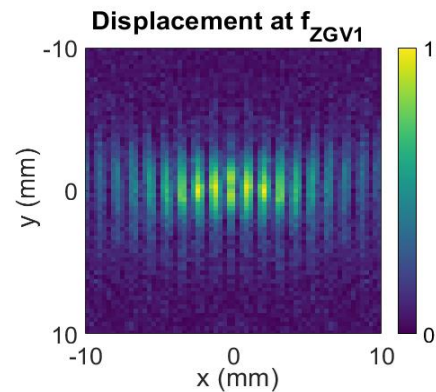
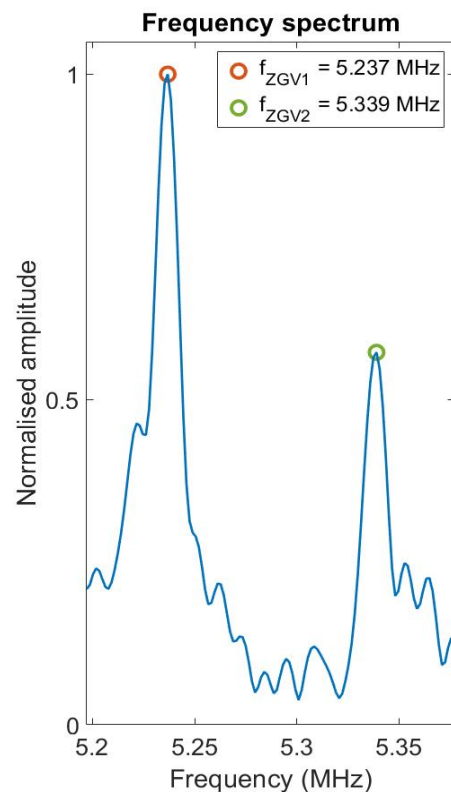
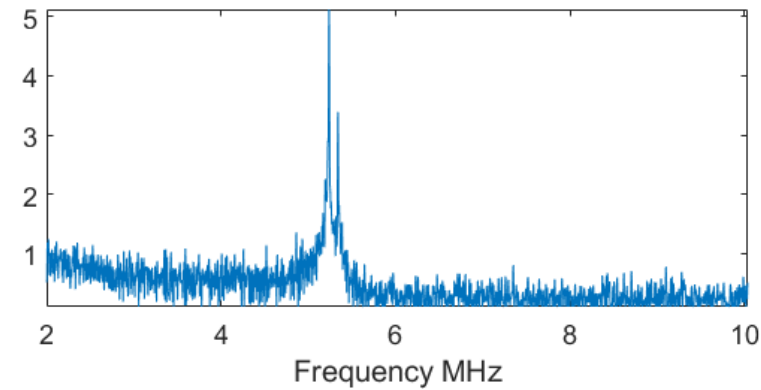
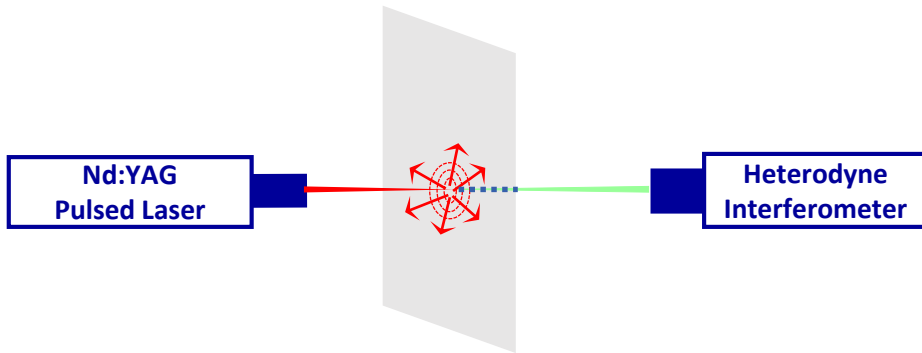
P = max. 0,045 %

S = max. 0,015 %

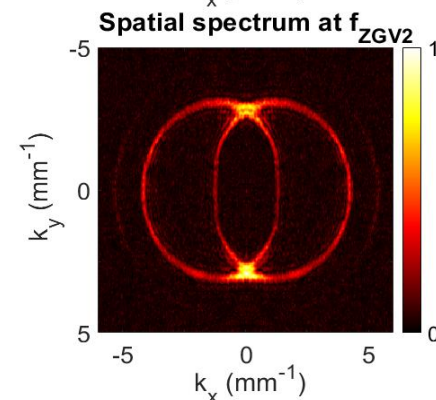
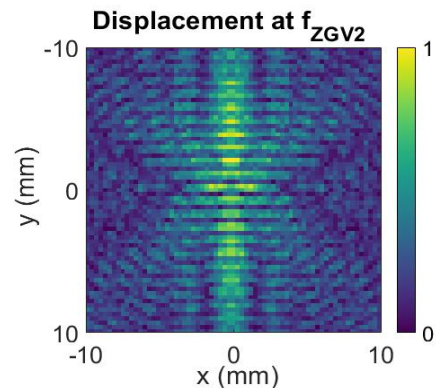
Cr = 16,0 – 19,0 %

Ni = 6,0 – 9,5 %

hasberg
SCHNEIDER GMBH



2 ZGV points
Minimum



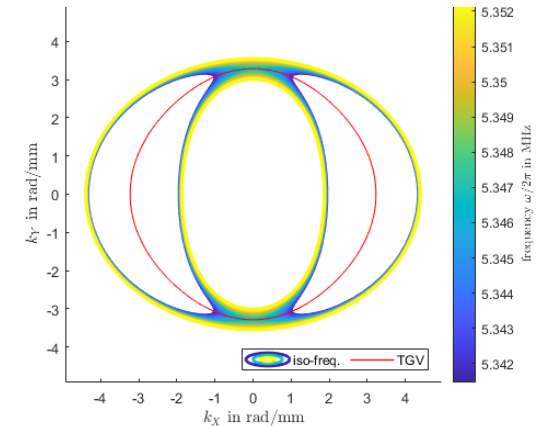
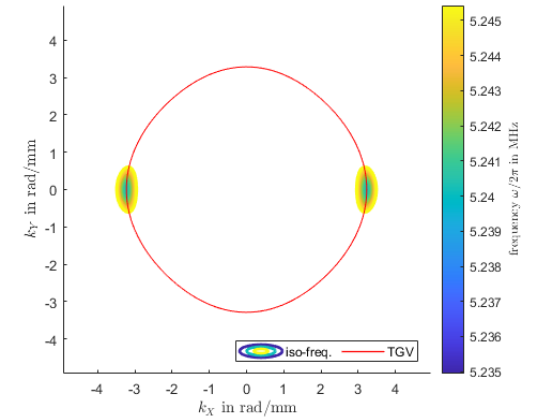
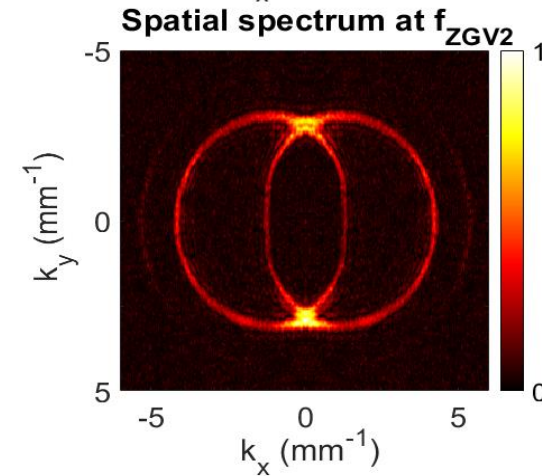
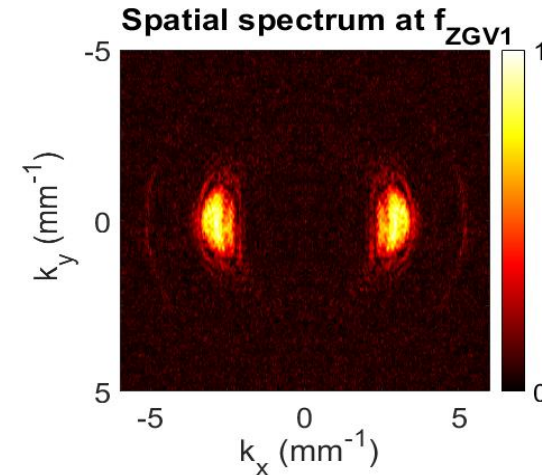
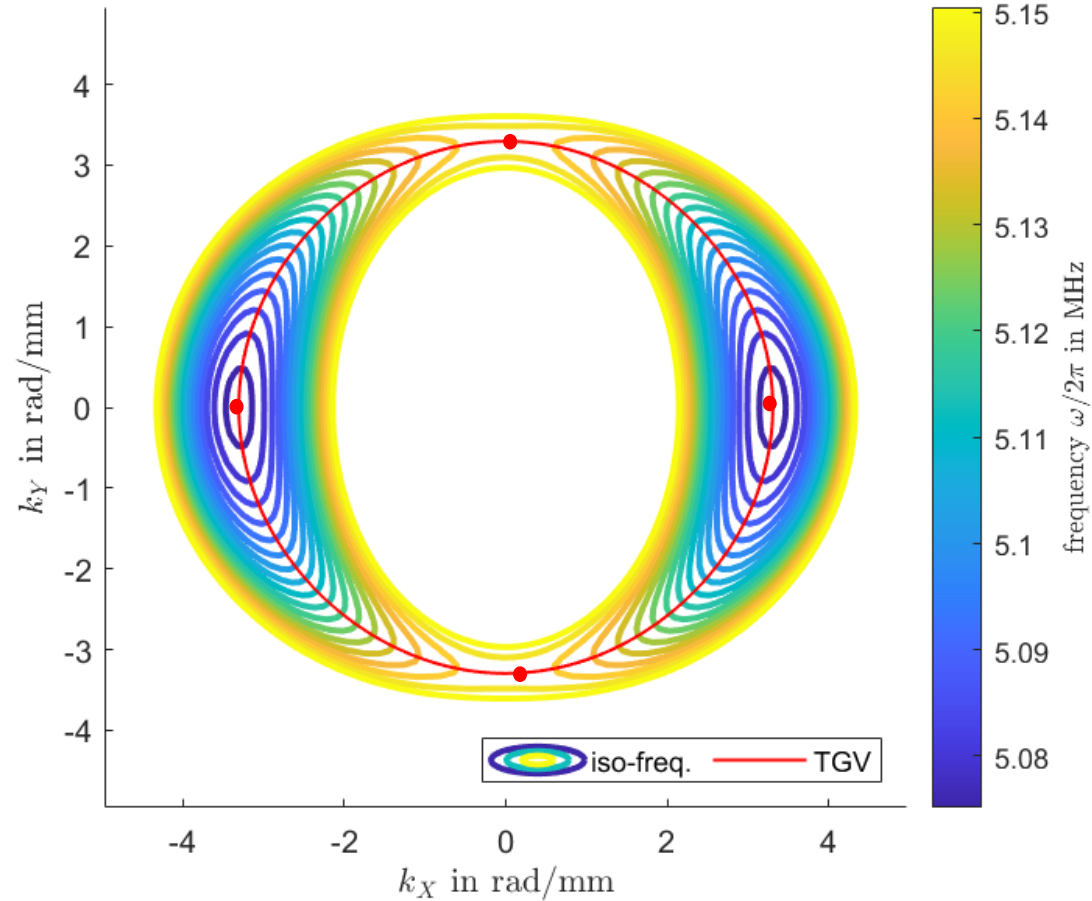
2 ZGV points
Saddle Point

Transverse Anisotropy



ZGV in a cold rolled stainless steel plate and dispersion surface

stainless steel 304 : S1/S2b mode

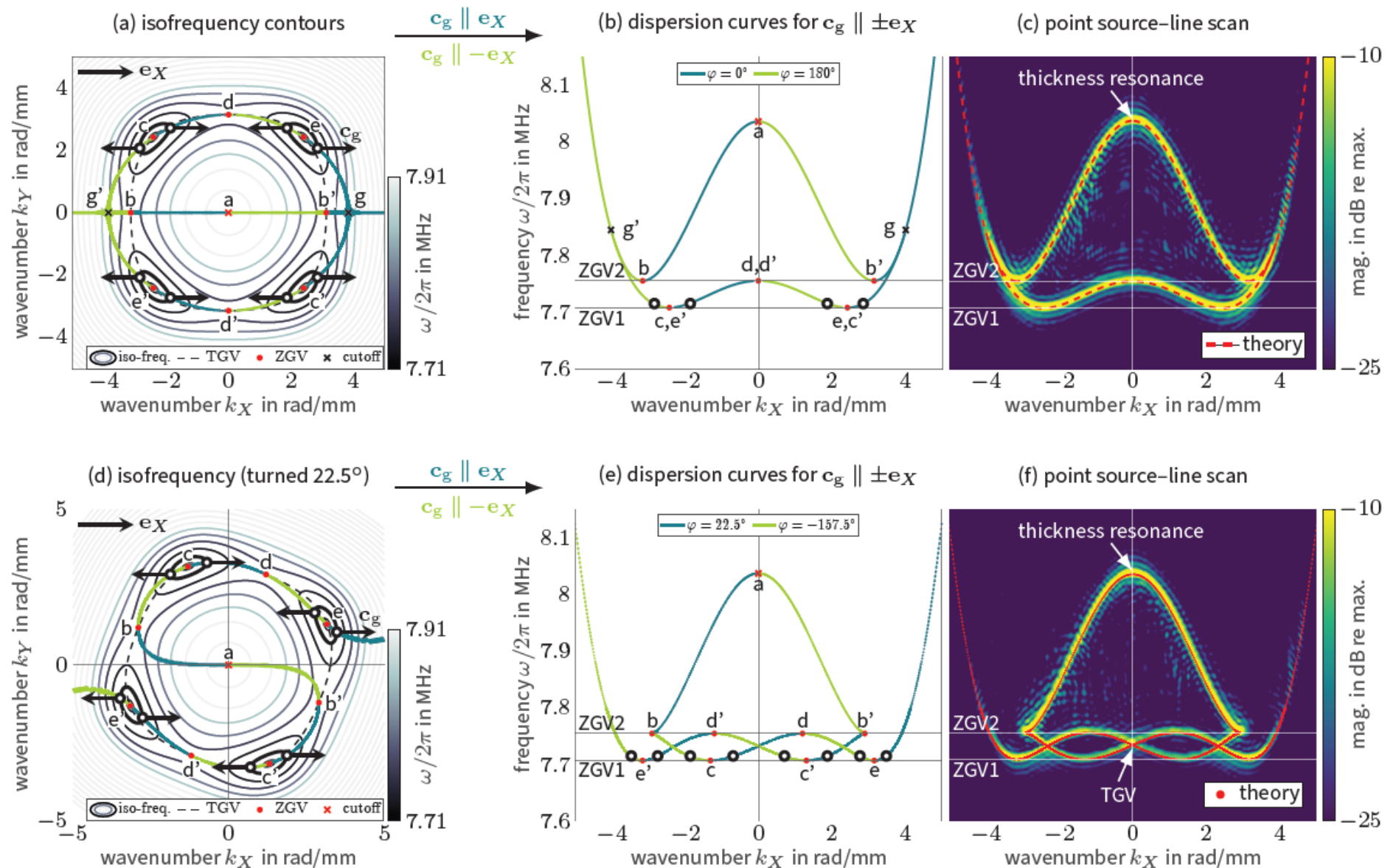


Transverse isotropic elastic constant
taken from
Laurent & al. JASA 137 (6) 2015

TO DO : built an inversion procedure



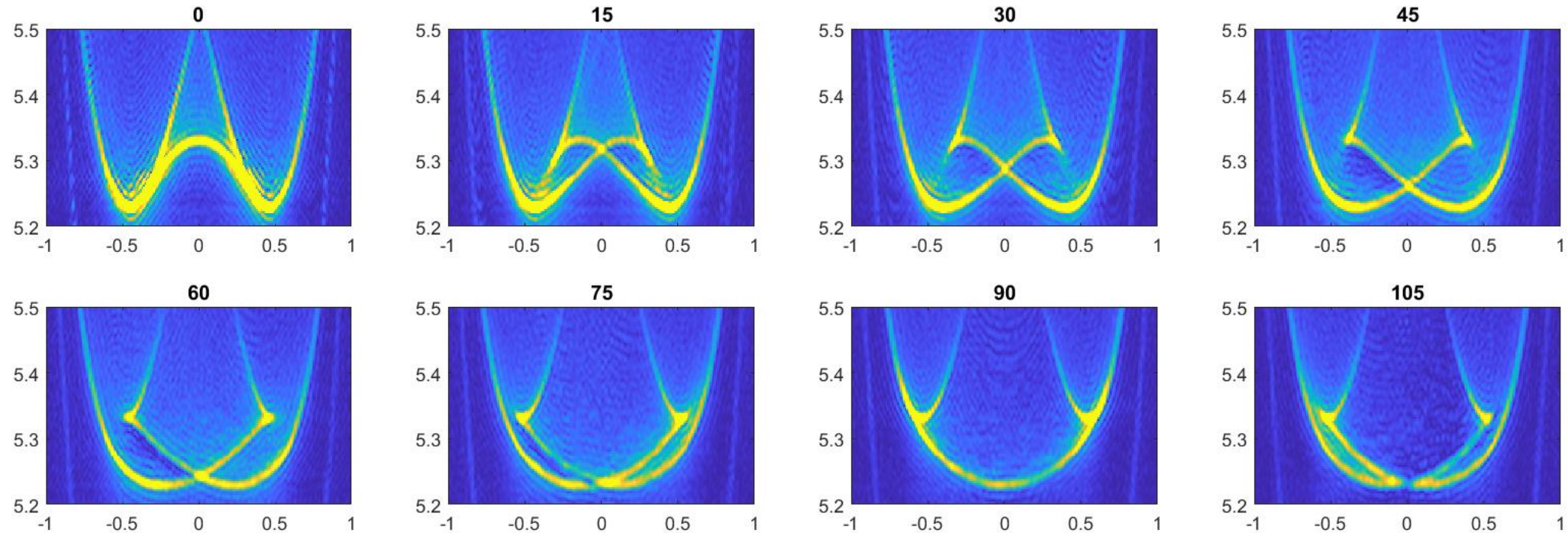
Dispersion curves from 1D scan in silicone





Dispersion curves from 1D scan in stainless steel 301

Line scan orientation 0 to 105 degrees by 15° steps



A 1D scan carries information from all propagation directions

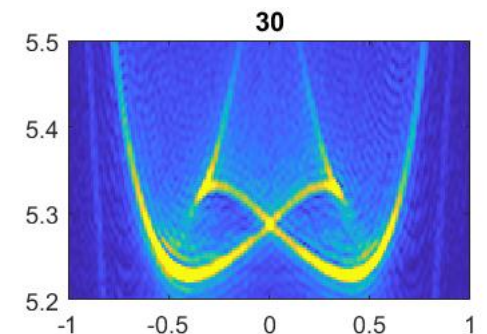
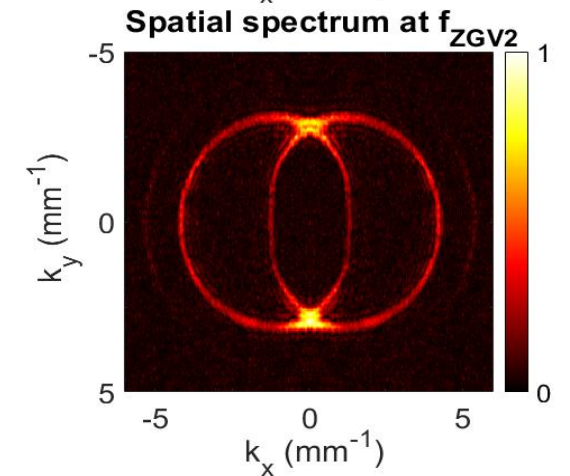
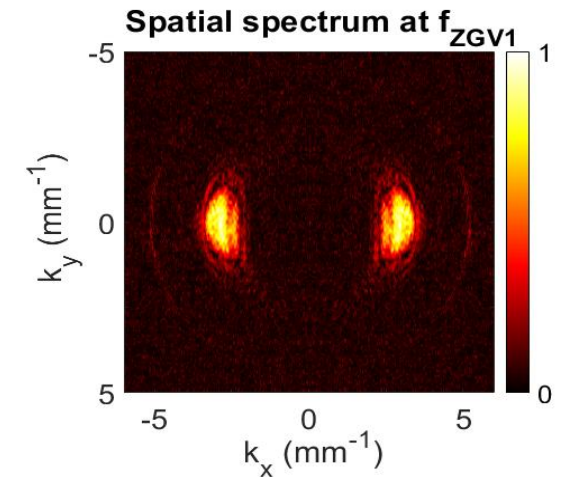
Would inversion be possible from a single line scan ?

Conclusion

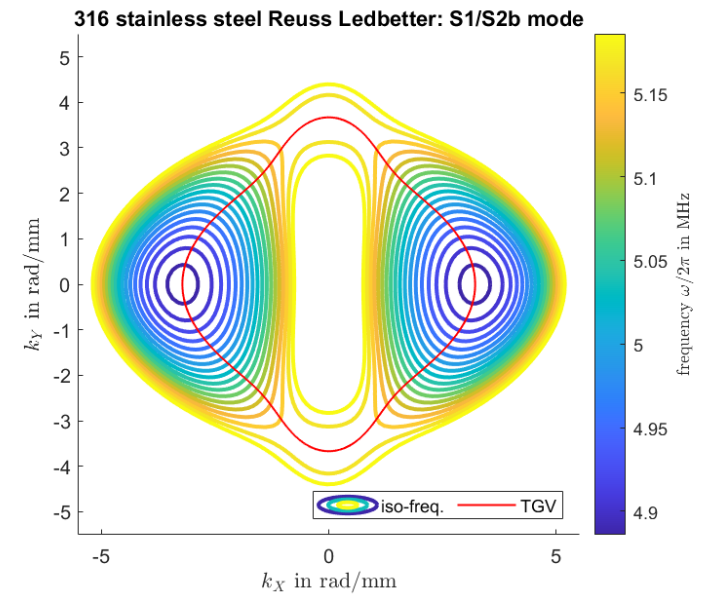
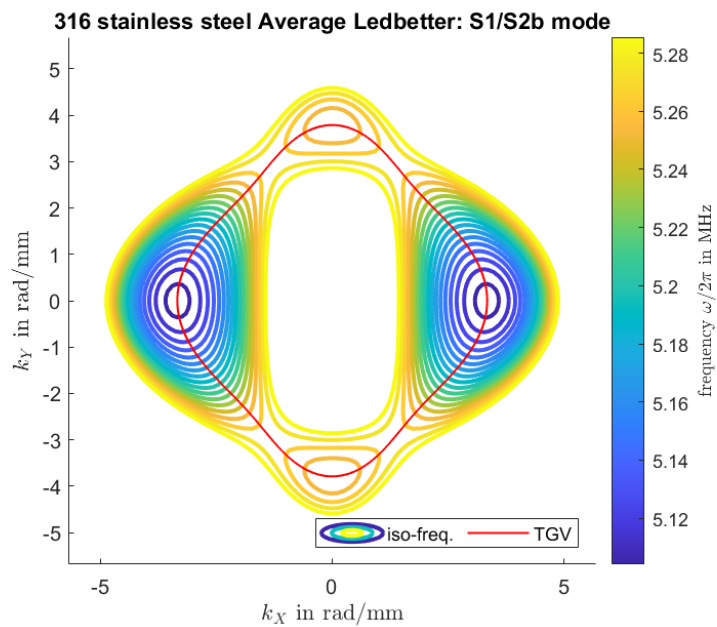
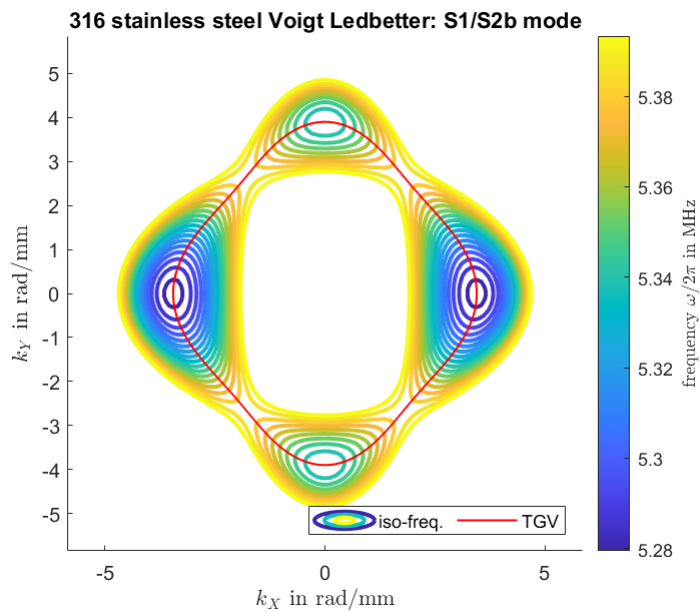
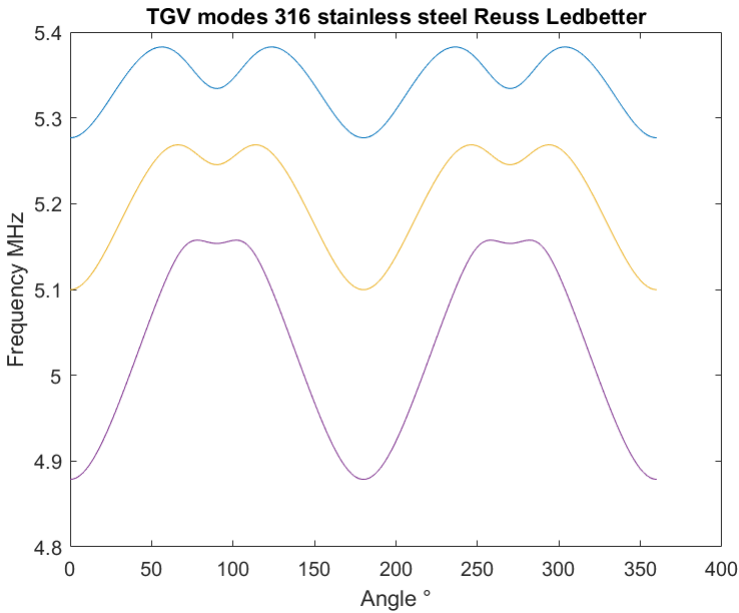
- ZGV mode resonances provide local non-contact evaluation of Poisson's ratio
- Anisotropic plates exhibit several ZGV resonance frequencies associated minima and saddle points of the same mode surface
- These resonances are highly sensitive to elastic constants
- Due to large beam skewing around ZGV points, 1D scan reveal several dispersion branches associated to same mode.

To do :

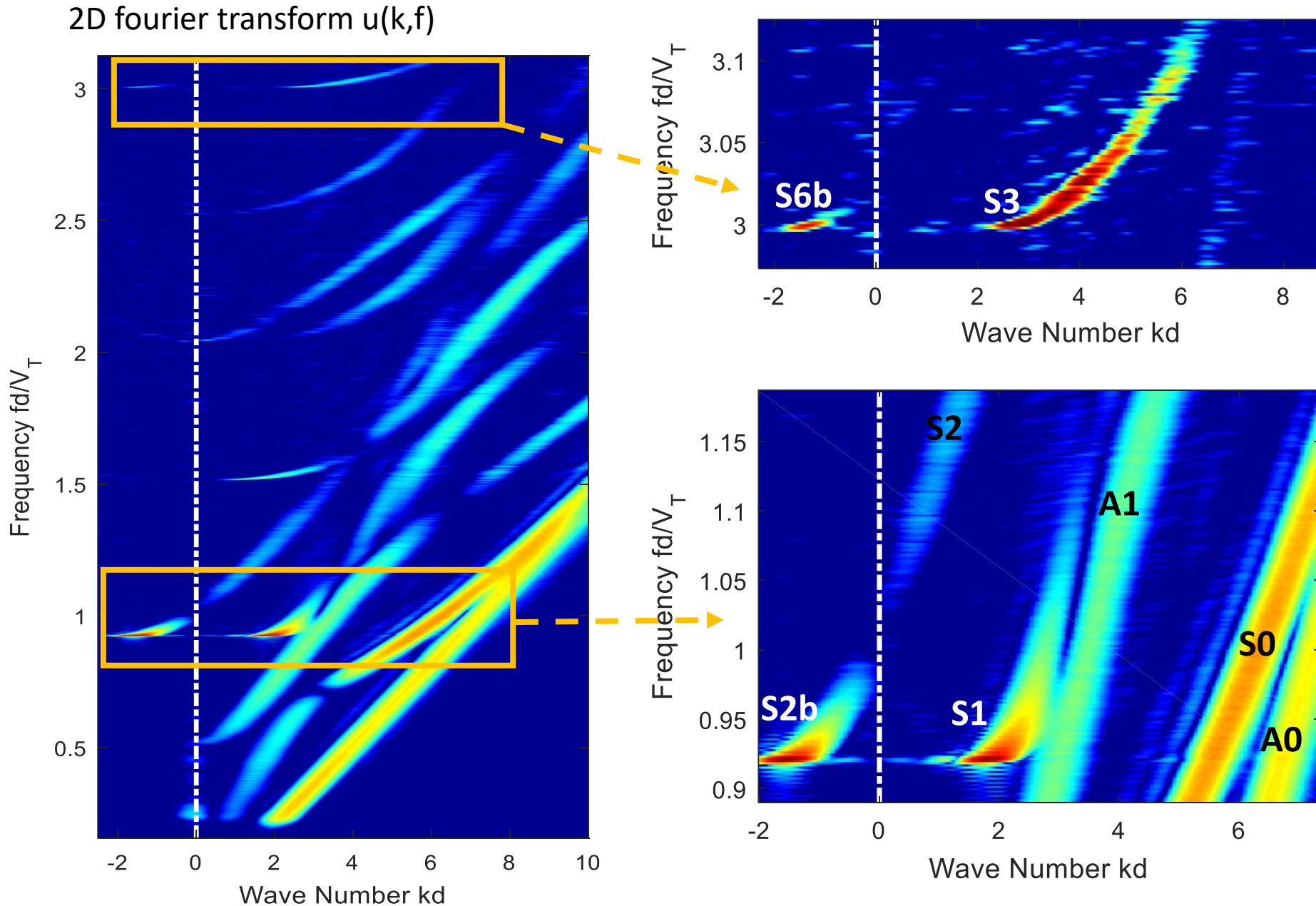
Inversion procedures for 2D and for 1D scan.



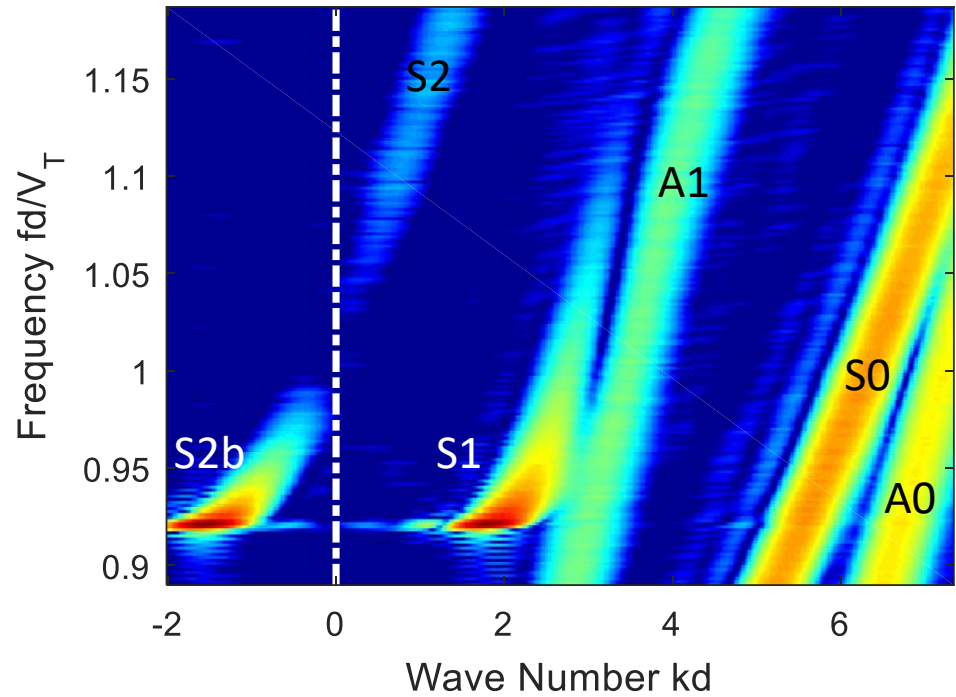
Constant taken from
Austenitic-steel elastic constants (MONOCRYSTAL/POLYCRYSTAL RELATIONSHIPS)
R. P. Reed et al. (eds.), *Austenitic Steels at Low Temperatures*
© Plenum Press, New York 1983



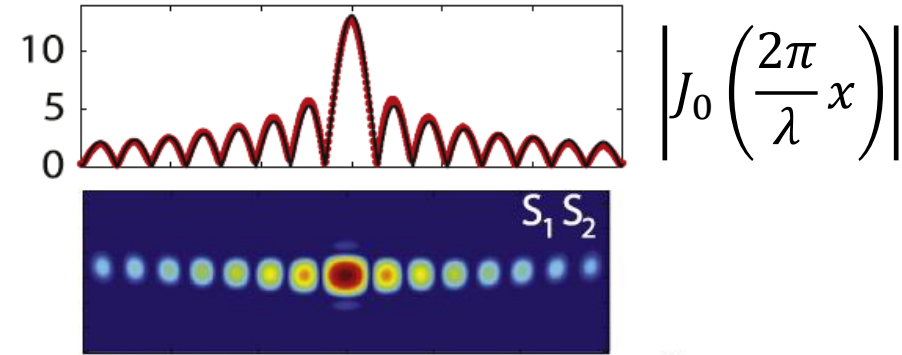
Measured dispersion curves of modes with positive group velocity



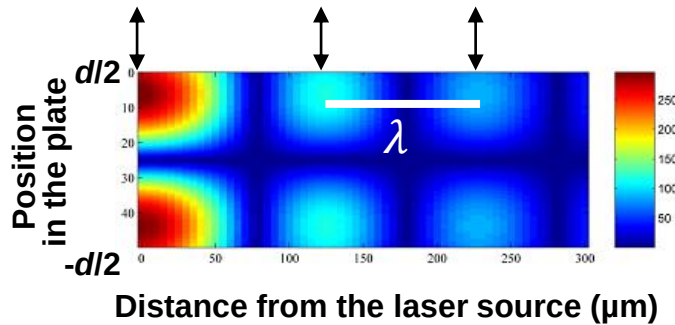
$S_1 S_2$ Zero Group velocity mode: interference of S1 and S2b mode



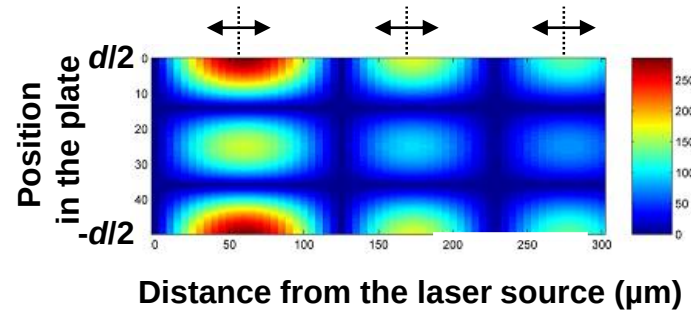
Out-of-plane surface displacement



Displacement inside the plate simulated with Spicer model (APL 1990)



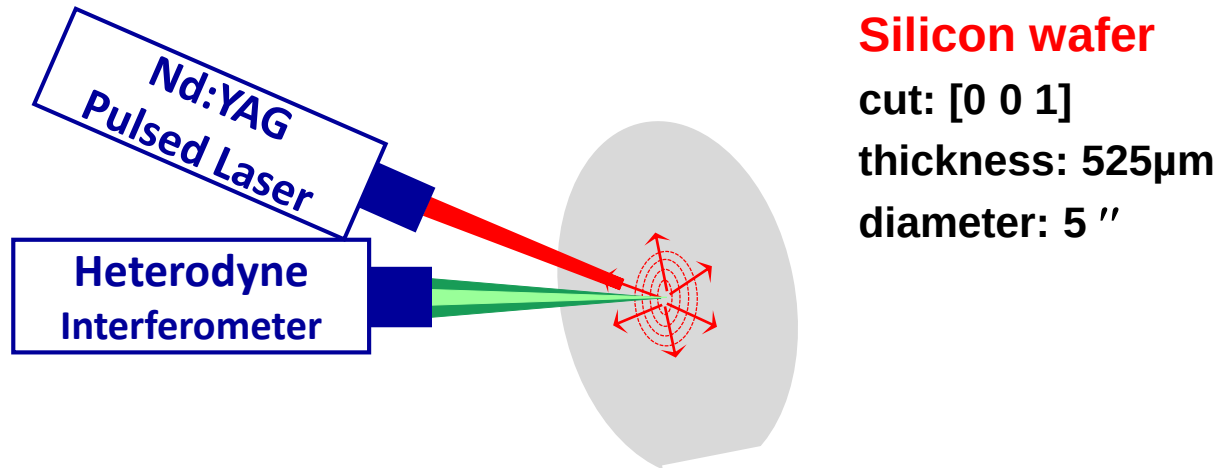
Out-of-plane displacement



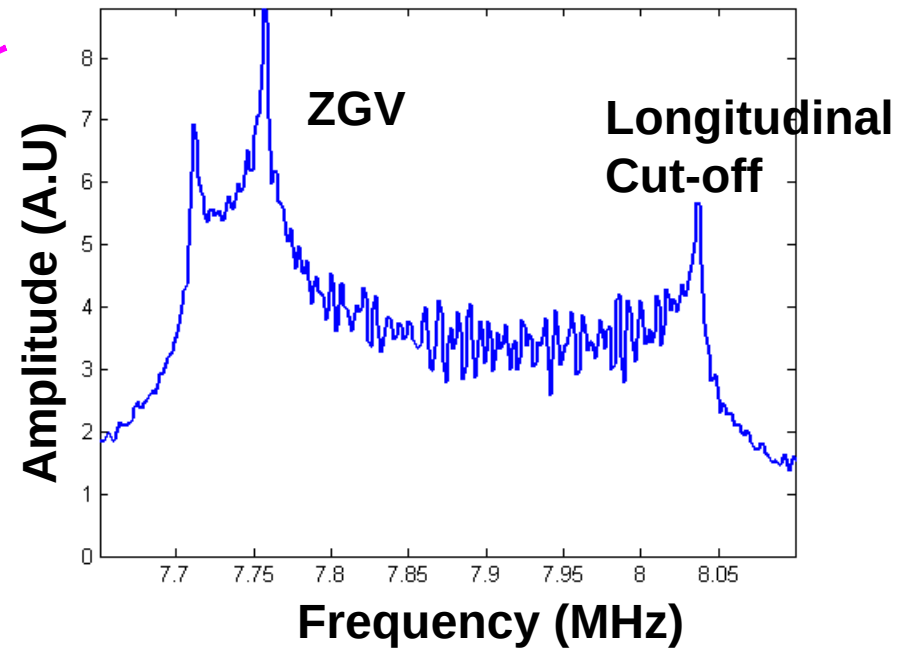
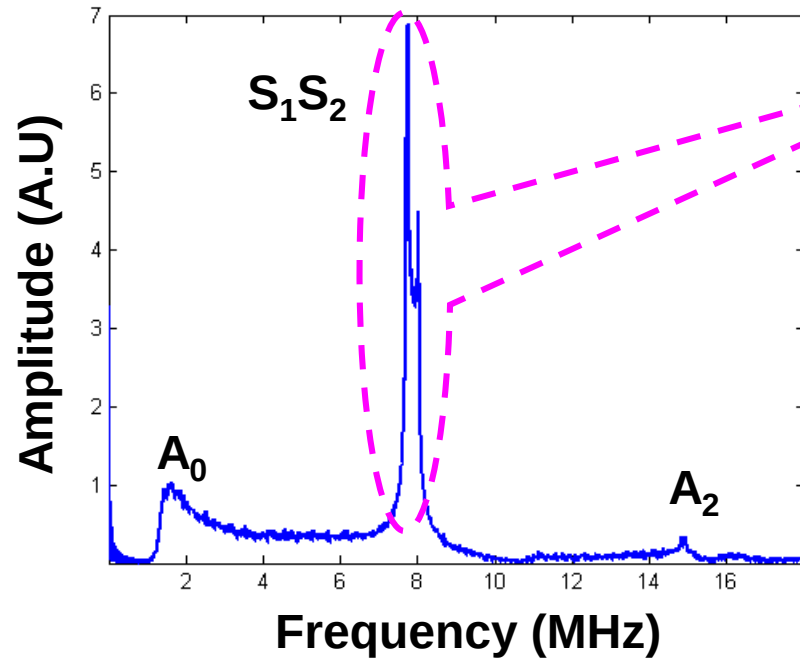
In-plane displacement



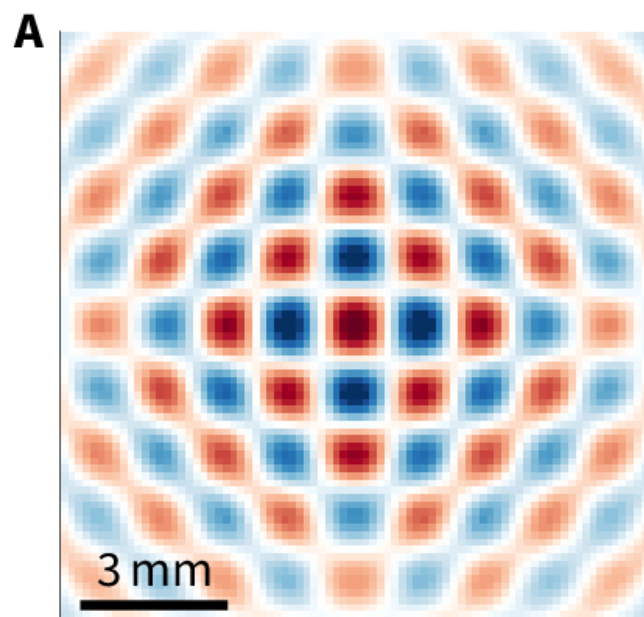
ZGV modes measured in a silicon plate



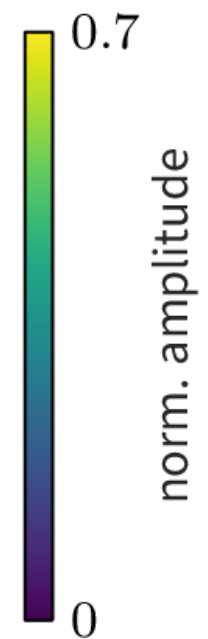
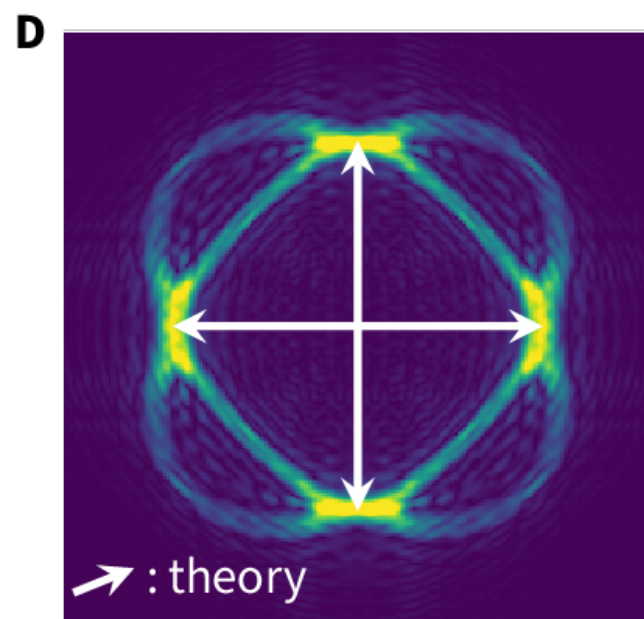
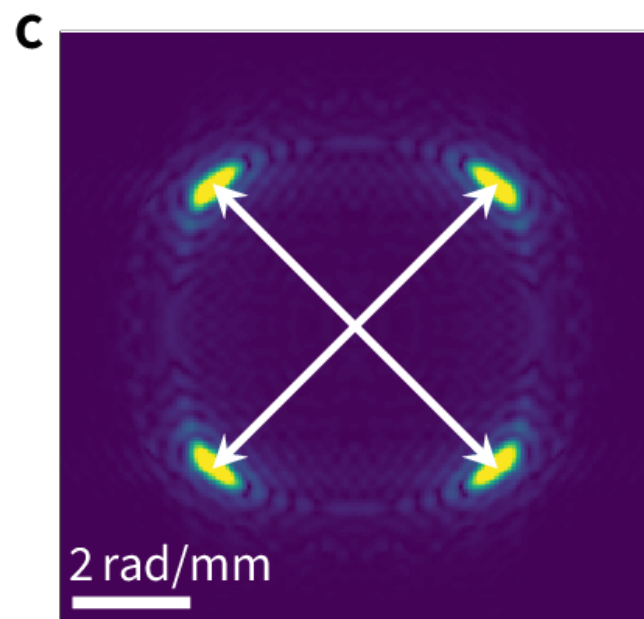
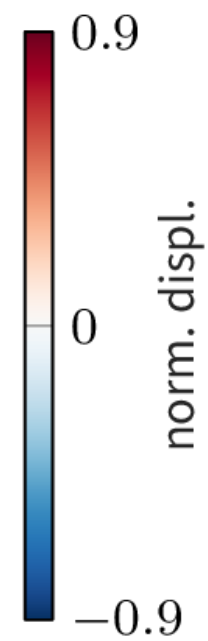
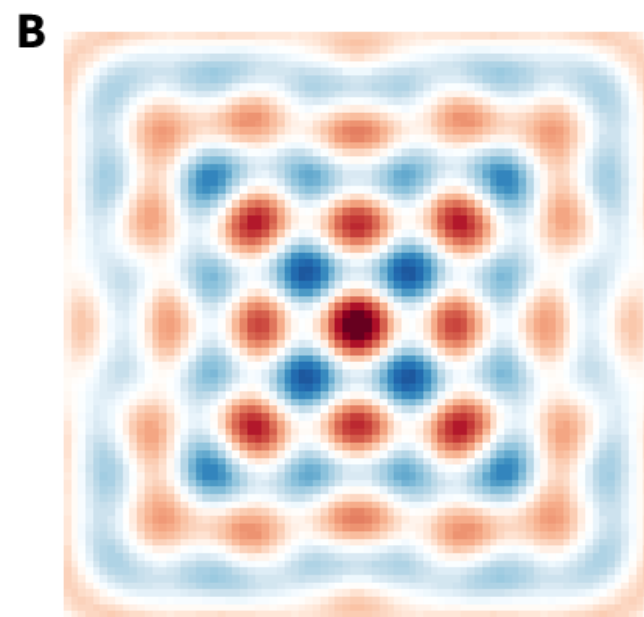
Normal displacement spectrum



7.7072 MHz (ZGV1)

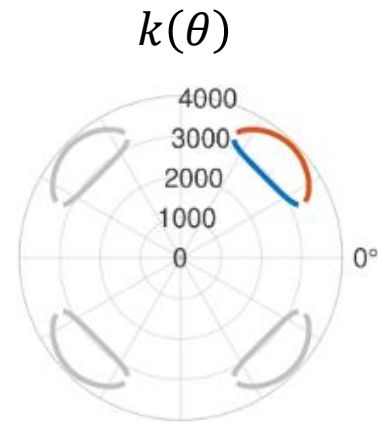


7.7545 MHz (ZGV2)

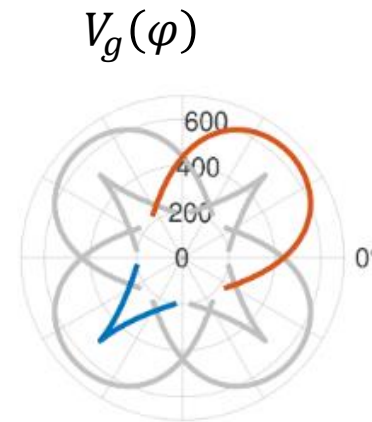


Experiment : below the saddle point frequency

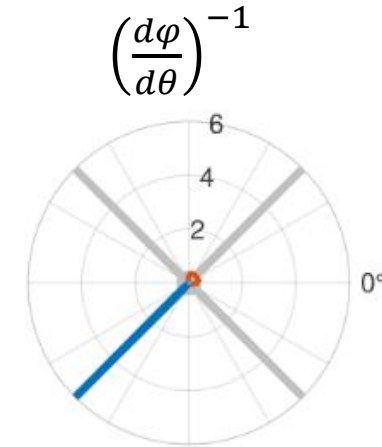
Slowness curves



Energy Velocity

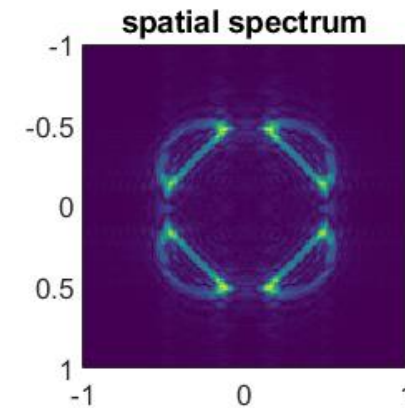
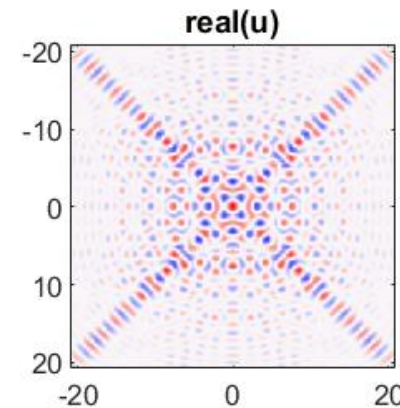
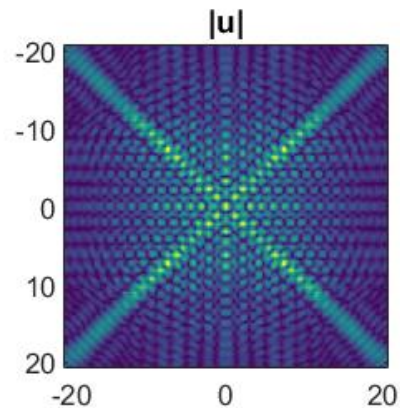
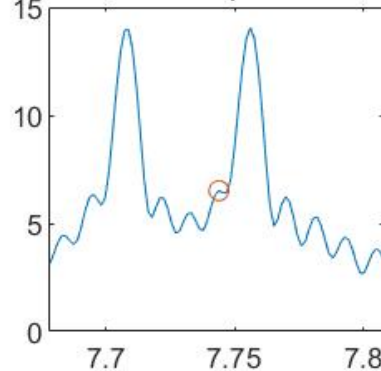


Maris Factor



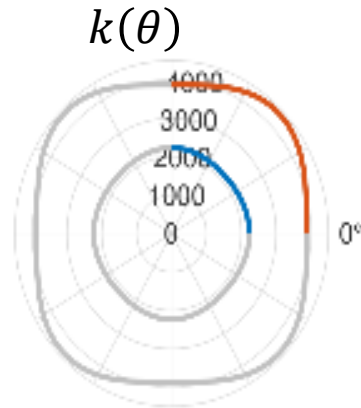
Fourier Transform of the measured field

$f = 7.7438 \text{ MHz}, 4.0 \text{ à } 94.0 \text{ } \mu\text{s}$

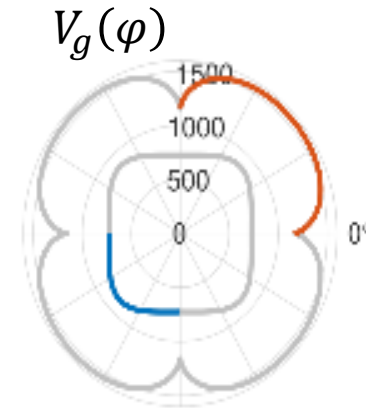


Experiment : above the saddle point frequency

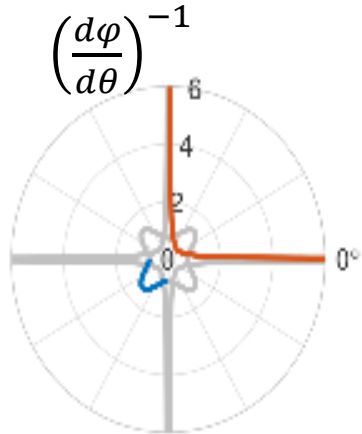
Slowness curves



Energy Velocity

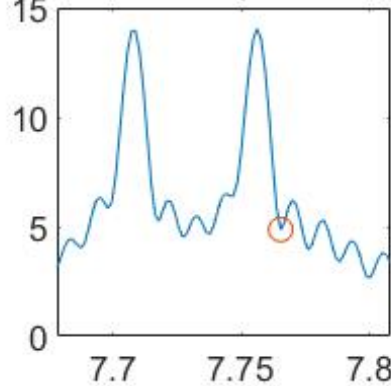


Maris Factor

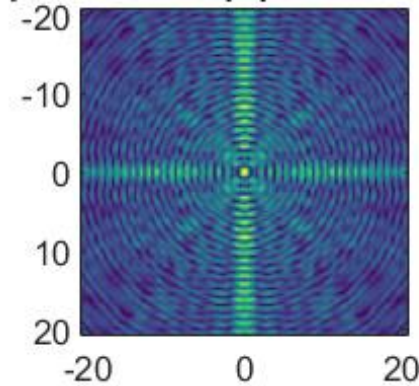


Temporal Fourier Transform of the measured field

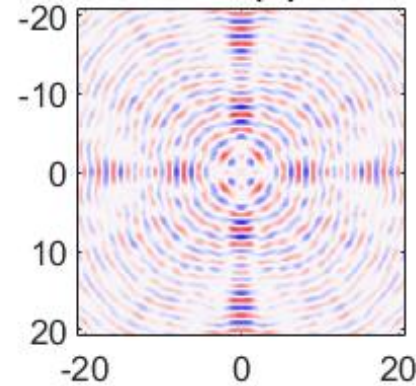
f = 7.7652 MHz, 4.0 à 94.0 μs



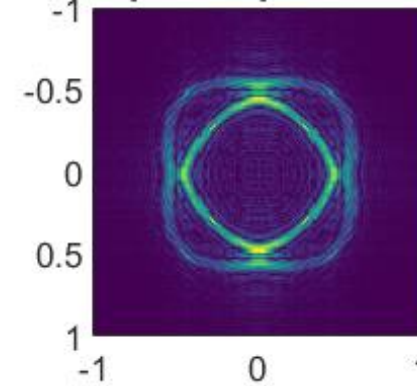
|u|



real(u)

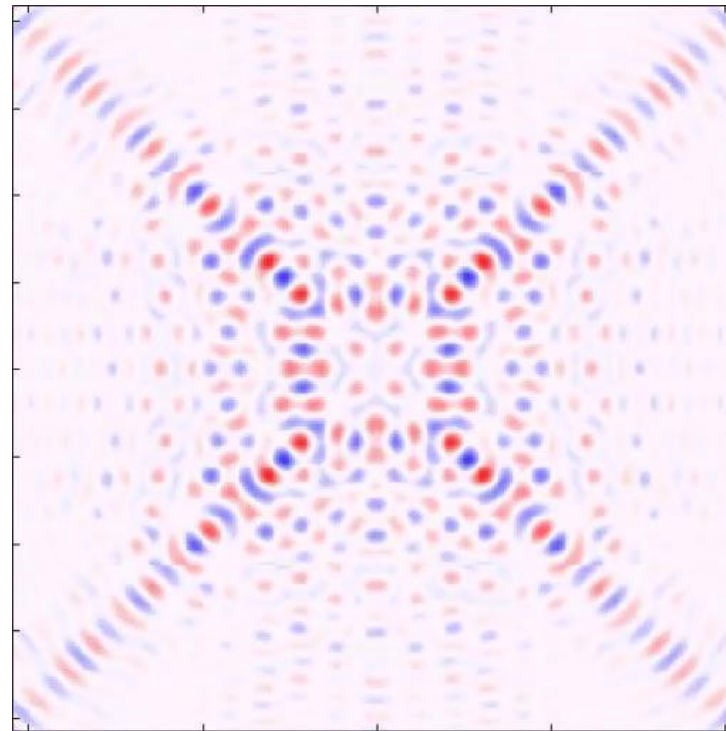
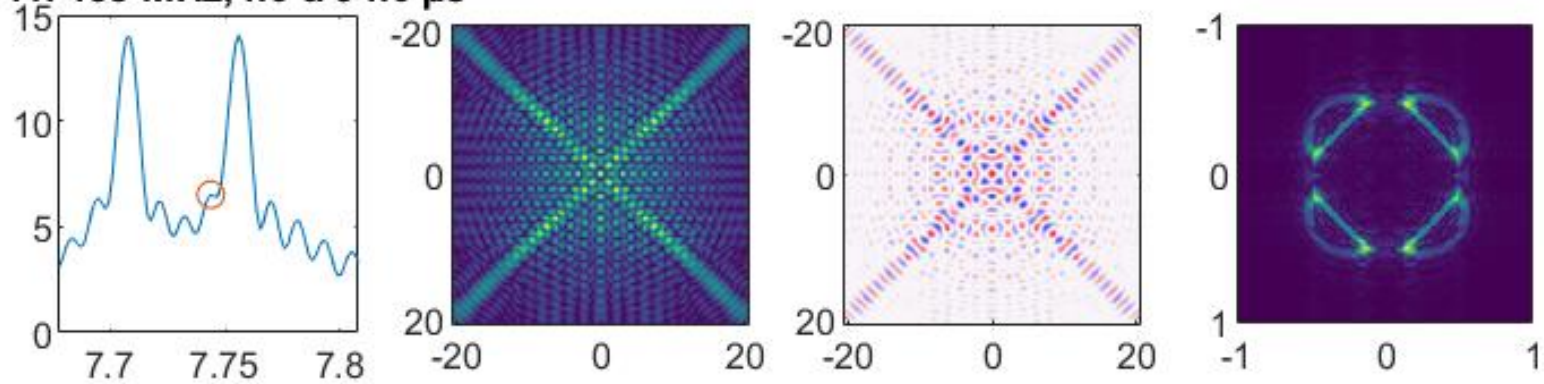


spatial spectrum



Backward Beam steering

$f = 7.7438 \text{ MHz}, 4.0 \text{ à } 94.0 \text{ } \mu\text{s}$



With Daniel Kiefer and Sylvain Mezil